ON THE DISTRIBUTION OF ALMOST PRIMES IN AN ARITHMETIC PROGRESSION

By

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1. Introduction. An almost prime is a positive integer the number of whose prime divisors is bounded by a certain constant. The purpose of this paper is to deal with an existence problem of almost primes in a short arithmetic progression of integers. We shall prove the following

Theorem. *Let k and l be two integers with $k \geqq 1$, $0 \leqq l \leqq k-1$, $(k, l)=1$. There exists a numerical constant $c_1 > 0$ such that for every real number $x \geqq c_1 k^{3.5}$ there is at least one integer n satisfying*

$$x < n \leqq 2x, \qquad n \equiv l \pmod{k}, \qquad V(n) \leqq 2,$$

where $V(n)$ denotes the total number of prime divisors of n. In particular, if we write $a(k, l)$ for the least positive integer n (>1) satisfying

$$n \equiv l \pmod{k}, \qquad V(n) \leqq 2,$$

then we have

$$a(k, l) < c_2 k^{3.5}$$

with some absolute constant $c_2 > 0$.

It is of some interest to compare our results presented above, though they are not the best possible, with a recent result of T. Tatuzawa [5] on the existence of a prime number p satisfying $x < p \leqq 2x$, $p \equiv l \pmod{k}$ and a celebrated theorem of Yu. V. Linnik concerning the upper bound for the least prime $p \equiv l \pmod{k}$ (cf. [3: X]).

Our proof of the theorem is based essentially upon the general sieve methods due to A. Selberg. The deepest result which we shall refer to is:

$$\pi(x) = \mathrm{li}\, x + O\left(x \exp(-c_3 (\log x)^{1/2})\right)$$

with a positive constant c_3, where $\pi(x)$ denotes, as usual, the number of primes not exceeding x (in fact, a slightly weaker result will suffice for our purpose). Apart from this, the proof is entirely elementary.

Notations. Throughout in the following, k represents a fixed positive