A UNIFIED THEOREM WITH SOME APPLICATIONS TO GENERALIZATIONS OF G. REEB'S THEOREM

By

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Introduction

The main purpose of the present paper is to bring into unity several theorems in the field of differential geometry. Some new results are included. The relevant theorems are cited below.

Theorem A. (de Rham [1]). *Let M be a simply-connected complete Riemannian manifold and let $T(M)$ be the tangent bundle with the holonomy group as structural group.*

If the Whitney sum of vector sub-bundles of $T(M)$:

$$T_1(M)+\cdots+T_p(M)$$

can be reduced to $T(M)$, then M turns out to be a product of Riemanian manifolds $M_1,\cdots,M_p$ and a Euclidean space E^d, namely

$$M=M_1\times\cdots\times M_p\times E^d.$$

Theorem B. (G. Reed [2] and J. Milnor [3]). *Let M be a compact differentiable manifold. If there exists a differentiable function with exactly two non-degenerate critical points over M, then M is homeomorphic to an n-sphere where n is the topological dimension of M.*

We shall state generalizations of this theorem at the end of this introduction.

Theorem C. (S. Kobayashi). (See [4]). *Let M be a complete Riemannian manifold. Then any Killing vector field defined over M generates a 1-parameter transformation group of isometries.*

Theorem D. (K. Nomizu [5]). *Let M be a simply-connected Riemannian manifold and let $\mathfrak{K}$ be the sheaf over M of germs of Killing vector fields. If dim $\mathfrak{K}_x$ do not depend on x, then $\mathfrak{K}$ is a constant sheaf.*

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*) The material of this paper comprises a portion of the author's doctoral dissertation, prepared under the direction of Prof. A. KAWAGUCHI.