

ON THE SPECTRUM OF FUNCTION IN THE WEYL SPACE

By

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1. Introduction. Let f be a bounded measurable function defined on the real line. By $\Lambda(f)$ we mean the set of real number λ which satisfies the following property. For any integrable function $K(x)$ the condition

$$(1.1) \quad K * f = \int_{-\infty}^{\infty} K(x-y)f(y)dy = 0 \quad (-\infty < x < \infty)$$

implies

$$(1.2) \quad \int_{-\infty}^{\infty} e^{i\lambda y} K(y) dy = 0.$$

(c. f. A. Beurling [1].)

In the previous paper we introduced the analogous definitions. By $\Lambda_*(f)$ we mean the set of real number λ which satisfies the following property. For any integrable function $K(x)$ the condition

$$(1.3) \quad K * f = \int_{-\infty}^{\infty} K(x-y)f(y)dy \sim 0$$

implies

$$(1.4) \quad \int_{-\infty}^{\infty} e^{i\lambda y} K(y) dy = 0$$

where the notation $K * f \sim 0$ means

$$(1.5) \quad \overline{\lim}_{l \rightarrow \infty} \sup_{-\infty < x < \infty} \frac{1}{l} \int_{-\infty}^{\infty} |K * f(x)|^2 dx = 0.$$

(c. f. S. Koizumi [4]. It is clear that

$$(1.6) \quad \Lambda_*(f) \subseteq \Lambda(f).$$

The purpose of this paper is to investigate properties of the above defined set as for functions represented by the Fourier-Stieltjes transform and almost periodic functions in the sense of Weyl.

2. General property of $\Lambda_*(f)$.

Theorem 1. *The set $\Lambda_*(f)$ is closed.*

This is a trivial result.