ON ENERGY INEQUALITIES OF MIXED PROBLEMS FOR HYPERBOLIC EQUATIONS OF SECOND ORDER

By

Rentaro AGEMI

§ 1. Introduction

Let $\boldsymbol{R}_+^n$ be the open half space $\{x=(x', x_n)\in \boldsymbol{R}^n;\ x'\in \boldsymbol{R}^{n-1},\ x_n>0\}$ and $\overline{\boldsymbol{R}}_+^n$ its closure. We consider a hyperbolic operator in $[0, T]\times \overline{\boldsymbol{R}}_+^n\ (T>0)$:

(1.1) $$P(t, x; D)=\frac{\partial^2}{\partial t^2}-2\sum_{j=1}^{n} a_j(t, x)\frac{\partial^2}{\partial t\partial x_j}-\sum_{j,k=1}^{n} a_{jk}(t, x)\frac{\partial^2}{\partial x_j\partial x_k}+(\textit{first order})$$

which satisfies

(1.2) $$\sum_{j,k=1}^{n}\left\{a_{jk}(t, x)+a_j(t, x)a_k(t, x)\right\}\xi_j\xi_k>0$$

(1.3) $$a_{nn}(t, x)>0$$

for any $(t, x)\in[0, T]\times \overline{\boldsymbol{R}}_+^n$ and any non-zero $\xi=(\xi_1, \cdots, \xi_n)\in \boldsymbol{R}^n$. The condition (1.2) means that $P(t, x; D)$ is strictly hyperbolic and (1.3) assures to impose *one* boundary condition on a mixed problem considered below (cf. §2). Here we assume that a_{jk} are symmetric. Moreover we consider the following boundary operator on the boundary $[0, T]\times(\overline{\boldsymbol{R}}_+^n-\boldsymbol{R}_+^n)$:

(1.4) $$B(t, x'; D)=\frac{\partial}{\partial x_n}-\sum_{j=1}^{n-1} b_j(t, x')\frac{\partial}{\partial x_j}-c(t, x')\frac{\partial}{\partial t}+h(t, x').$$

Here we assume that all coefficients in (1.1) and (1.4) are real valued, sufficiently smooth and constant except a compact set.

In the case of operators with constant coefficients, a necessary and sufficient condition for L^2-well-posedness[1)] of a mixed problem with homo-

1) The mixed problem (P, B) is L^2-well-posed if and only if there exist positive constants C, T and T' $(0<T'\leq T)$ satisfying the following property: For every $f\in H^1((0, T)\times \boldsymbol{R}_+^n)$ with $f=0$ $(t<0)$ the problem

$$Pu=f\ (t>0,\ x_n>0),\ Bu=0\ (t>0,\ x_n=0),\ u=\frac{\partial u}{\partial t}=0\ (t=0,\ x_n>0)$$

has a unique solution $u\in H^2((0, T')\times \boldsymbol{R}_+^n)$ such that

$$\int_0^{T'}|||u(t,\cdot)|||_1^2\,dt\leq C\int_0^{T}\|f(t,\cdot)\|_0^2\,dt.$$