# On homotopically trivial links

Dedicated to Professor Yoshie Katsurada on her 60th birthday

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## §1. Introduction.

Throughout this paper we shall only be concerned with the combinatorial category, consisting of simplicial complexes and piecewise linear maps (for the combinatorial categoryf see [3]). Zeeman [3] shows that if $(n, k, 2)$-*link* $L=(S^n \supset K_1^k \cup K_2^k)$ is homotopically trivial and if $2n \geqq 3k+4$, then $L$ is geometrically trivial. And if $n=k+2$, it is well known that there exists a homopically trivial $(n, n-2, 2)$-*link* which is not geometrically trivial (for example, in $(3, 1, 2)$-*link*). For the case $n \geqq k+3$ and $2n \leqq 3k+3$ Zeeman says that there exist homotopically trivial links which are not geometrically trivial same as $(3, 1, 2)$-*link*. But there is no proof for these links be geometrically non-trivial. So we consider the relation between homotopically trivial links and geometrically trivial links under $n \geqq k+3$ and $2n \leqq 3k+3$. In this paper we obtain a geometrical sufficient condition for a homotopically trivial $(n, k, 2)$-*link* be geometrically trivial. I should like to express my sincere gratitude to the members of Kōbe and Hokkaidō topology seminars for many discussion of this problem.

## §2. Notations and Definitions

$S^n$ is a standard $n$-sphere and $D^n$ is a standard $n$-cell. An $(n, k, 2)$-*link* $L$ is a pair $(S^n \supset K_1^k \cup K_2^k)$ of an $n$-sphere $S^n$ and a disjoint union of locally flatly embedded $k$-spheres $K_i^k$, $i=1, 2$ in $S^n$. $\partial X$ and $\mathrm{Int}\, X$ mean the boundary and the interior of a manifold $X$. $X * Y$ denote the join of spaces $X$ and $Y$. $\cong$ means "homeomorphic to". $\overset{m}{\underset{i=1}{V}} S_i^p$ means one point join of $p$-spheres $S_1^p, \cdots, S_m^p$ and $\mathrm{I}=[0, 1]$. For any manifolds $X$, $Y$ such that $X$ is a submanifold of $Y$, $U(X, Y)$ means a regular neighborhood of $X$ in $Y$. And we always take $U(X, Y)$ to be a second derived neighborhood of $X$ in $Y$ for a suitable subdivision of $X$ and $Y$ unless otherwise stated.

DEFINITION 1. Let $X$, $Y$ be subsets in an $n$-manifold $Z$. We say that $X$ and $Y$ *split each other* in $Z$ if there exists an $n$-cell $B^n$ in $Z$ such that either $X \subset \mathrm{Int}\, B^n$, $Y \cap B^n = \phi$ or $Y \subset \mathrm{Int}\, B^n$, $X \cap B^n = \phi$.