A remark on the Steenrod representation of $B(Z_p \times Z_p)$

Dedicated to Professor Yoshie Katsurada on her 60th birthday

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§ 1. Introduction

For a topological space X, $z \in H_n(X; Z)$ is Steenrod representable if there exists a closed oriented smooth n-manifold M and a continuous map $f: M \to X$ such that $f_*(\sigma)=z$, where σ is a fundamental homology class of M. In [4], Thom showed that for a finite polyhedron X any $z \in H_n(X; Z)$ is representable if $n \leqq 6$, but if $n \geqq 7$ not everything is representable. He exhibited a class in $H_7(L^7(3) \times L^7(3); Z)$ which was not, where $L^7(3)$ is 7-dimesional lens space mod 3. Moreover Burdick [1] extended to $B(Z_3 \times Z_3)$, classifying space of $Z_3 \times Z_3$, and computed all representable elements. He dermined E^∞ terms of bordism spectral sequence of $B(Z_3 \times Z_3)$ and used necessary condition of representability of Thom [4].

In this note we show the case $p=2$ and any odd prime p. Latter case we use the same methods as Burdick's.

We have

THEOREM 1.

(*a*) *Every elements of* $H_*(B(Z_2 \times Z_2); Z)$ *are Steenrod representable.*

(*b*) *For p an odd prime the elements of* $H_*(B(Z_p \times Z_p); Z)$ *which are Steenrod representable are generated by* $e_0 \otimes e_0$, $e_{2i-1} \otimes e_{2j-1}$, $e_0 \otimes e_{2j-1}$, $e_{2i-1} \otimes e_0$, $\{(e_2 \otimes e_{2j-1} + e_1 \otimes e_{2j}) + (e_6 \otimes e_{2j-5} + e_5 \otimes e_{2j-4}) + \cdots\}$, *and* $\{(e_4 \otimes e_{2j-3} + e_3 \otimes e_{2j-2}) + (e_8 \otimes e_{2j-7} + e_7 \otimes e_{2j-6}) + \cdots\}$.

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§ 2. Homology groups of $B(Z_p \times Z_p)$

Let $X = B(Z_p \times Z_p)$, $Y = B(Z_p)$.

Case (a): $p=2$.

Let RP^n be the n dimensional real projective space, RP^∞ be the direct limit of it. Then we can consider $Y = RP^\infty$, and so $X = Y \times Y$. The cell structure of RP^n and its boundary operations are given as follows:

$$RP^n = e_0 \cup e_1 \cup \cdots \cup e_n,$$