Generalized Minkowski formulas for closed hypersurfaces in a Riemannian manifold

Dedicated to Professor Yoshie Katsurada on her Sixtieth Birthday

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Introduction.

Our starting point is the following well known formula for an ovaloid F in an Euclidean space E^3 of three dimensions:

$$\iint_F (Kp+H)\,dA=0, \tag{0. 1}$$

where H and K are the mean curvature and the Gauss curvature at a point P of F, p denotes the oriented distance from a fixed point 0 in E^3 to the tangent space of F at P and dA is the area element of F at P. For convex hypersurfaces, these formulas have been obtained by H. Minkowski for $m=2$ **[11]**[1] and by T. Kubota for a general m **[9]** (cf. also **[2]**, p. 64).

As a generalization of this formula for a closed orientable hypersurface, C. C. Hsiung derived the following integral formulas of Minkowski type which are valid in an Euclidean space E^{m+1} **[4]**.

THEOREM A (C. C. Hsiung) *Let V^m be a closed orientable hypersurface twice differentiably imbedded in an Euclidean space E^{m+1} of $m+1$ ($\geqq 3$) dimensions, then*

$$\int_{V^m} H_{\nu+1}p\,dA+\int_{V^m} H_\nu\,dA=0 \ \textit{for}\ \nu=0,1,\cdots,m-1, \tag{0. 2}$$

where $H_0=1$, $H_\nu (1\leqq\nu\leqq m)$ be the ν-th mean curvature of V^m at P, p denotes the oriented distance from a fixed point 0 in E^{m+1} to the tangent hyperplane of V^m at P, and dA be the area element of V^m at P.

Extension of this formulas in a Riemannian manifold R^{m+1} has been established by Y. Katsurada **[5]** **[6]**. Main result for a hypersurface V^m in a Riemannian manifold is as follows:

THEOREM B (*Y. Katsurada*) *Let V^m be a closed orientable hypersurface of class C^3 imbedded in an $(m+1)$-dimensional Riemannian manifold R^{m+1} which admits an infinitesimal point transformation, then*

1) Numbers in brackets refer to the references at the end of the paper.