# On Riemannian Manifolds Admitting a Certain Transformation

Dedicated to Professor Yoshie Katsurada on her 60th birthday

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## § 0. Introduction.

Let $(M^n, g)$ $(n \geqq 2)$ be an $n$-dimensional Riemannian manifold[1] with a positive definite metric tensor $g$. The following is well known.

THEOREM A (T. Nagano [7][2]). *Let $(M^n, g)$ $(n \geqq 4)$ be a Riemannian manifold. If $C$ does not vanish on $(M^n, g)$, there exists a metric tensor $g'$ conformal to $g$ such that*

$$C(M^n, g) = I(M^n, g')$$

*where $C$ is the Weyl's conformal curvature tensor of $(M^n, g)$, and $C(M^n, g)$ and $I(M^n, g)$ are the groups of conformal transformations and isometric transformations of $(M^n, g)$, respectively.*

From Theorem A, we can conjecture that $(M^n, g)$ is conformally flat if $C(M^n, g) \neq I(M^n, g)$ ([10]). In this respect, we can consider that if $(M^n, g)$ admits a transformation with a certain property, it shall restrict some geometrical properties of $(M^n, g)$. Taking the above into consideration, we shall study groups of homothetic transformations, conformal transformations and projective transformations.

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## § 1. Homothetic transformations.

Let $C(M^n, g)$, $A(M^n, g)$, $H(M^n, g)$ and $I(M^n, g)$ be groups of conformal transformations, affine transformations, homothetic transformations and isometric transformations of $(M^n, g)$, respectively. M. S. Knebelman and K. Yano ([4], [5], [13]) proved the followings.

THEOREM B (K. Yano). *In a compact Riemannian manifold $(M^n, g)$*

1) Throughout the paper, we assume that $(M^n, g)$ is connected.

2) The numbers in brackets refer to references at the end of this paper.