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Local solution of Cauchy problem for nonlinear hyperbolic systems in Gevrey classes

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Introduction

The Cauchy problem for nonlinear hyperbolic equations in Gevrey classes was studies by Leray-Ohya [7] (c. f. [8]). They assume that the characteristics are of constant multiplicity or smooth. In this paper we shall remove this restriction.

We consider the following equations for the unknowns $u(x)=(u_1(x), \cdots, u_N(x))$, $x=(x_0, x_1, \cdots, x_n)=(x_0, x')\in R^{n+1}$,

$$F_i\left(x, D^{M_i}u(x)\right)=0 \text{ in } \Omega, \qquad i=1, \cdots, N, \tag{0. 1}$$

where Ω is a neighborhood of 0 in R^{n+1} and

$$D^{M_i}u(x)=\left\{D^{M_{i1}}u_1(x), \cdots, D^{M_{iN}}u_N(x)\right\}$$

$$D^{M_{ij}}u_j(x)=\left\{(\partial/\partial x_0)^{\alpha_0}(\partial/\partial x_1)^{\alpha_1}\cdots(\partial/\partial x_n)^{\alpha_n}u_j(x)\,;\ \alpha=(\alpha_0, \alpha_1, \cdots, \alpha_n)\in M_{ij}\right\}$$

and M_{ij} is a finite set of non negative multi indices.

We assume that $\{F_i\}$ is a Leray-Volevich system of order m, that is, there exist non negative integers $n_1, \cdots, n_N$ such that for $\alpha\in M_{ij}$,

$$|\alpha|=\alpha_0+\alpha_1+\cdots+\alpha_n\leqslant m+n_j-n_i, \qquad i, j=1, \cdots, N. \tag{0. 2}$$

Then we can prescribe the following Cauchy data to the equations (0. 1),

$$(\partial/\partial x_0)^j u_i(0, x')=\varphi_{ji}(x'), \qquad j=0, \cdots, m-1,\ i=1, \cdots, N. \tag{0. 3}$$

We introduce coordinate variables

$$y_{ij}=(y_\alpha\,;\ \alpha\in M_{ij}) \text{ in } R^{r_{ij}}, \qquad i, j=1, \cdots, N,$$

$$y_i=(y_{ij}\,;\ j=1, \cdots, N) \text{ in } R^{r_i}, \qquad i=1, \cdots N,$$

$$y=(y_i\,;\ i=1, \cdots, N) \text{ in } R^r,$$

where r_{ij} is the number of the elements of M_{ij}, $r_i=r_{i1}+\cdots+r_{iN}$ and $r=r_1+\cdots+r_N$.

We assume that $F_i(x, y)$, $i=1, \cdots, N$ are in Gevrey class s in x and