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Analytic wavefront sets and operators with multiple characteristics

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0. Introduction

In this paper we study the analytic wavefront set of solutions of certain differential equations with multiple characteristics. In many situations one can obtain very general results in the analytic theory while the corresponding results in the C^{∞} theory are much more complicated. As an example we can mention the very general theorem about propagation of analytic singularities for microhyperbolic operators, due to Kashiwara-Kawai [8]. This result only requires a hyperbolicity assumption on the principal symbol of the operator. (The results in the C^{∞} theory are more complicated, less complete, and depend in general on the lower order symbols of the operator). In [14], [15] we developped some methods to handle situations where only the principal symbol has to be considered. A common point in the various proofs is the inversion of a suitable elliptic problem. Here we will study problems where a reduction to an elliptic problem seems impossible (at least sometimes), and instead we study perturbations of the given problem which are non-elliptic but for which suitable a priori estimates can be obtained.

To be more specific, the class of operators that we shall study (in the analytic case) is the one introduced by Boutet de Monvel, Grigis, Helffer [2]. These authors obtained a very general and satisfactory result concerning the C^{∞}-hypoellipticity with minimal loss of derivatives. On the other hand Trèves [19], Tartakoff [17] and more generally G. Metivier [11] proved the analytic regularity when the characteristic variety is symplectic. We shall give a new proof of Metiviers result. The readers conclusion will hopefully be that the analytic regularity in this case is an easy consequence of facts which are *essentially* known from the C^{∞}-theory. We will also give some new results in non-symplectic situations. An interesting feature here is the use of Lagrangian manifolds which are only of Lipschitz class. (If we had restricted the attention to Metivier's theorem a shorter proof could certainly have been given). We will also give an extension of a theorem of Ôaku [13]. We beleive that the general method of this paper can and will be