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The Cauchy problem for effectively hyperbolic operators

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1. Introduction

Following the pioneering work of Oleinik [8], Ivrii & Petkov [5] and then Hörmander [2] analysed the structure of the principal symbol of a weakly hyperbolic differential operator near a double characteristic. In particular, Ivrii & Petkov conjectured that if the only multiple characteristics are double and these are effectively hyperbolic, in the sense that the Hamilton map has a non-zero real eigenvalue, then the Cauchy problem is well-posed for any lower order terms; that is, strongly well-posed in C^{∞} in the sense of Ivrii [3]. In this paper the well-posedness modulo C^{∞} of the Cauchy problem is proved using microlocal energy estimates derived for an analogously defined class of effectively hyperbolic pseudodifferential operators. More restricted results in this direction have been obtained, under additional hypotheses, by Oleinik [8], who dealt with operators in two independent variables and whose work was subsequently extended by Nishitani, by Ivrii [4] who employed a form of the principal symbol corresponding to a separation of variables and by Iwasaki [6] who assumed a slighly weaker, but still non-trivial, condition of Poisson commutation on the principal symbol see also Yoshikawa [9]. The necessity of effective hyperbolicity for the strong well-posedness in C^{∞} of the Cauchy problem for a differential operator with only double characteristics was shown by Ivrii & Petkov [5].

To define the notion of an effectively hyperbolic pseudodifferential operator, let $P\in\Psi_{cl}^{m}(X)$ be a classical pseudodifferential operator on the C^{∞} manifold X and suppose that P has real principal symbol $p\in C^{\infty}(T^{*}X\backslash 0)$. If $\bar{\rho}\in T^{*}X\backslash 0$ is a double characteristic for P:

$$p(\bar{\rho})=0\,,\qquad dp(\bar{\rho})=0$$

the Hessian of p at $\bar{\rho}$ is well-defined:

$$\mathrm{Hess}\,(p):\ T_{\bar{\rho}}M\times T_{\bar{\rho}}M\rightarrow\boldsymbol{R}\,,\qquad M=T^{*}X\,.$$

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