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On the hyperbolicity in the domain of real analytic functions and Gevrey classes

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§. 1 Introduction

We are concerned with the Cauchy problem for the following first order equation

$$(1.1)\qquad \partial_t u=\sum_{j=1}^{l} A_j(x, t)\,\partial_x u+B(x, t)\,u+f,$$

where $x=(x_1, \cdots, x_l)\in \boldsymbol{R}^l$, $t\in \boldsymbol{R}$; $u(x, t)={}^t(u_1(x, t), \cdots, u_m(x, t))$, and $A_j(1\leqq j\leqq l)$ and B are matrices of order m. All the coefficients are assumed to be real analytic in x and continuous in t.

The Cauchy-Kowalewsky theorem, more precisely the Nagumo-Ovcianni-kov theorem asserts that, given any real analytic initial data $\varphi(x)\in C^{\omega}(\mathscr{O}_x)$ and $f(x, t)\in C_t^0(C^{\omega}(\mathscr{O}_x))$ (continuous function of t with values in $C^{\omega}(\mathscr{O}_x)$), where $\mathscr{O}_x(\subset \boldsymbol{R}^l)$ is an open connected neighborhood of the origin.

We are concerned with the existence domain of u. Let $f=0$. Then its domain may depend on the initial data φ, more precisely on its radius of convergence around the origin. However, the Bony-Schapira theorem asserts that, when A_j and B are analytic in (x, t), and if the characteristic roots $\lambda_i(x, t;\xi)$ of

$$(1.2)\qquad \det\left(\lambda I-\sum_{j} A_j(x, t)\,\xi_j\right)=0$$

are all real, then there exists a neighborhood of the origin, say V, such that for any $\varphi(x)\in C^{\omega}(\mathscr{O}_x)$, there exists a unique solution $u(x, t)\in C^{\omega}(V)$. It is plausible that this result can be extended to the actual situation. Our aim is to show that

THEOREM 1. *If there exists a common existence domain V of the solution $u(x, t)$ for any real analytic initial data $\varphi(x)\in C^{\omega}(\mathscr{O}_x)$, then the characterustuc roots $\lambda_i(x, 0;\xi)$ $(1\leqq i\leqq m)$ should be real.*

In §. 6, we shall explain what becomes Theorem 1 in the case of the class s of Gevrey $(1<s<+\infty)$. Concerning this subject, there are two