On some hypersurfaces satisfying $R(X, Y)\cdot R_1 = 0$

Dedicated to Professor Yoshie Katsurada on her sixtieth birthday

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1. Introduction.

The Riemannian curvature tensor R of a locally symmetric Riemannian manifold (M, g) satisfies

(*) $\qquad R(X, Y)\cdot R = 0 \qquad$ for all tangent vectors X and Y,

where the endomorphism $R(X, Y)$ operates on R as a derivation of the tensor algebra at each point of M. Conversely, does this algebraic condition (*) on the curvature tensor field R imply that $\nabla R = 0$? K. Nomizu conjectured that the answer is positive in the case where (M, g) is complete, irreducible and dim $M \geqq 3$. But, recently, H. Takagi [5] gave an example of 3-dimensional complete, irreducible Riemannian manifold (M, g) satisfying (*) and $\nabla R \neq 0$. Moreover, the present author proved that, in an $(m+1)$-dimensional Euclidean space E^{m+1} $(m \geqq 3)$, there exist some complete, irreducible hypersurfaces which satisfy the condition (*) and $\nabla R \neq 0$. For example,

(1. 1) $$M;\quad x_{m+1} = (x_1 - x_2)^2 x_2 + (x_1 - x_2) x_3 + \sum_{a=1}^{m-3} x_{a+3} e^{a(x_1 - x_2)} \qquad m \geqq 4,$$

(1. 2) $$M;\quad x_4 = (x_1 - x_2)^2 x_2 + (x_1 - x_2) x_3, \qquad \text{(See [3])},$$

(1. 3) $$M;\quad x_4 = \frac{x_1^2 x_3 - x_2^2 x_3 - 2 x_1 x_2}{2(1 + x_3^2)}, \qquad \text{(See [5])},$$

where $(x_1, x_2, \cdots, x_{m+1})$ denotes a canonical coordinate system on E^{m+1}.

By these examples, we see that K. Nomizu's conjecture is negative. For theses examples, we see that the type number $k(x)$ is at most 2 for each point $x \in M$ and actually 2 at some point of M. In [2], K. Nomizu proved

THEOREM A. *Let (M, g) be an m-dimensional complete Riemannian manifold which is isometrically immersed in E^{m+1} so that the type number $k(x) \geqq 3$ at least at one point $x \in M$. If (M, g) satisfies the condition (*), then it is of the form $S^k \times E^{m-k}$, where S^k is a hypersphere in a Euclidean subspace E^{k+1} of E^{m+1} and E^{m-k} is a Euclidean subspace orthogonal to E^{k+1}.*