On Beurling's theorem

By Hiroshi Tanaka*)

Introduction

Let \( R, R' \) be hyperbolic Riemann surfaces and \( \phi \) be an analytic mapping of \( R \) into \( R' \). Let \( K_0 \) be a closed disk in \( R \) and let \( R_0 = R - K_0 \). Let \( C \) be the Kuramochi capacity on \( R_0 \cup \Delta_N \) and \( \Delta_i \) be the set of all minimal Kuramochi boundary points of \( R \). For a metrizable compactification \( R'^* \) of \( R' \), we denote by \( \mathcal{A}(\phi) \) the set of all points in \( \Delta_i \) at which \( \phi \) has a fine limit in \( R'^* \). There are two typical extensions of Beurling's theorem [1] to analytic mappings of a Riemann surface to another one, i.e., Z. Kuramochi's [5, 6, 7] and C. Constantinescu and A. Cornea's theorems [3, 4]. The former result states that if \( \phi \) is an almost finitely sheeted mapping and \( R'^* \) is H.D. separative, then \( \mathcal{C}(\Delta_i - \mathcal{A}(\phi)) = 0 \). The latter one states that if \( \phi \) is a Dirichlet mapping and \( R'^* \) is a quotient space of the Royden compactification of \( R' \), then \( \mathcal{C}(\Delta_i - \mathcal{A}(\phi)) = 0 \). The present author [9] proved that these two results are independent. In this paper we shall give another extension of Beurling's theorem such that it contains the above two results: If \( \phi \) is a Dirichlet mapping and \( R'^* \) is H.D. separative, then Beurling's theorem is valid.

Notation and terminology

Let \( R \) be a hyperbolic Riemann surface. For a subset \( A \) of \( R \), we denote by \( \partial A \) and \( A^\epsilon \) the (relative) boundary and the interior of \( A \) respectively. We call a closed or open subset \( A \) of \( R \) is regular if \( \partial A \) is non-empty and consists of at most a countable number of analytic arcs clustering nowhere in \( R \). We fix a closed disk \( K_0 \) in \( R \) once for all and let \( R_0 = R - K_0 \).

1. Function spaces and compactifications (cf. [4]).

We denote by \( BC = BC(R) \) the space of all bounded continuous (real-valued) functions on \( R \). Let \( BCW = BCW(R) \) be (resp. \( BCD = BCD(R) \)) the family of all bounded continuous Wiener functions (resp. bounded continuous Dirichlet functions) on \( R \). It is known ([4]) that both \( BCW \) and \( BCD \) are vector sublattices of \( BC \) with respect to the maximum and minimum opera-