A relation between the fibers of Milnor fiberings associated to polynomials $f(z)=z_0^{a_0}+\cdots+z_n^{a_n}$

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§ 0. Introduction

All manifolds will be oriented and differentiable of class C^∞. Let $a=(a_0, a_1, a_2, \cdots, a_n)$ be a set of integers, $a_i>1$ and consider a polynomial $f(z_0, z_1, \cdots, z_n)=z_0^{a_0}+z_1^{a_1}+\cdots+z_n^{a_n}$, $z_i \in \boldsymbol{C}$ $(i=0, 1, 2, \cdots, n)$. If K_a is the intersection of $f^{-1}(0)$ and the unit sphere S^{2n+1} in $\boldsymbol{C}^{n+1}$, then we have an associated Milnor fibering ϕ: $S^{2n+1}-K_a \rightarrow S^1$. It is well known that a fiber F_a of ϕ is a $(n-1)$-connected 2n-manifold and the closure $\bar{F}_a$ of F_a in S^{2n+1}, a manifold with boundary K_a (see [5]). The purpose of this paper is to give a relation between F_a and F_b, where b is another set of integers, $b=(b_0, b_1, \cdots, b_n)$, $a_i \leqq b_i$ $(i=0, 1, 2, \cdots, n)$.

By Pham's results [6] we can give a canonical basis $x_1, x_2, \cdots, x_{\mu_a}$ to H_n $(\bar{F}_a; \boldsymbol{Z})$ and also a basis $y_1, y_2, \cdots, y_{\mu_b}$ to H_n $(\bar{F}_b: \boldsymbol{Z})$, where $\mu_a=(a_0-1)(a_1-1)\cdots(a_n-1)$ and $\mu_b=(b_0-1)(b_1-1)\cdots(b_n-1)$. (see Theorem 1.6). Then we have

THEOREM A. *Let* F_a, F_b, $\{x_i\}_{i=1,2,\cdots,\mu_a}$ *and* $\{y_j\}_{j=1,2,\cdots,\mu_b}$ *be as above. If* $n\geqq 3$, *then there exists a smooth embedding* e: $\bar{F}_a \rightarrow \bar{F}_b$ *so that each* x_i *is mapped onto* y_i *by* $(e)_*$ *and that* $\bar{F}_b-e(F_a)$ *is a manifold with boundary* $(-K_a)\cup K_b$ $(i=0, 1, \cdots, \mu_a)$.

This is proved by considering the intersection pairing of $H_n(\bar{F}_a)$, $H_n(\bar{F}_b)$ and maps α: $\pi_n(\bar{F}_a)$ (and $\pi_n(\bar{F}_b)) \rightarrow \pi_{n-1}(SO_n)$ which are defined in [7].

Let $a=(2, 2, \cdots, 2, s)$. If s odd, then we have well known results that K_a is a homotopy sphere which is determined in [1]. But if s is even, then K_a is not a homotopy sphere. As an application of Theorem A we have the following

THEOREM B. i) *If n is even, then* K_a *is diffeomorphic to* $D^n \times S^{n-1} \cup_{f_a} S^{n-1} \times D^n$, *where* f_a *is described as follows. Let* ∂: $\pi_n(S^n) \rightarrow \pi_{n-1}(SO_n)$ *be a boundary homomorphism associated to the fibration* $SO_n \rightarrow SO_{n+1} \rightarrow S^n$, $\iota_n = id_{S^n}$ *and* $\varphi_a=\partial([s/2]\,\iota_n)$. *Then a diffeomorphism* f_a: $S^{n-1}\times S^{n-1} \rightarrow S^{n-1}\times S^{n-1}$ *is given by* $f_a(x, y)=(x, \varphi_a(x), y)$.

ii) *If n is odd, then* K_a *is diffeomorphic to*