On the Kuramochi boundary of a subsurface of a Riemann surface

By Yukio NAGASAKA

Introduction

Z. Kuramochi [4] considered a compactification of a subsurface G of a Riemann surface R, which is similar to the Kuramochi compactification of R. In [4], he introduced the function-theoretic mass for F_0SH functions on G and showed that it is important to investigate the properties of the compactification of G. Then, on the subsurface G, there are two topologies: one of them is the topology on the compactification of G and the other is the induced topology on G by the Kuramochi compactification of R. Z. Kuramochi [3] investigated the relations of these two topologies (Theorem A and B).

In this paper, we shall give some properties of the function-theoretic mass (Proposition 2 and Theorem 1) and that the F_0H function with finite function- theoretic mass is represented by the canonical measure (Theorem 2). In §5, we shall study the relation of the above two topologies (Theorem 4, 5 and 6).

§1. Notation and terminology

Let R be a hyperbolic Riemann surface. We call a closed or open subset A of R regular if the relative boundary ∂A of A consists of at most a countable number of analytic arcs clustering nowhere in R. We fix a closed disk K_0 in R and a regular subdomain G of R such that $K_0 \cap G = \phi$. Let $R_0 = R - K_0$. An exhaustion of R will mean an increasing sequence $\{R_n\}$ of relatively compact domains on R such that $\bigcup_{n=1}^{\infty} R_n = R$ and each ∂R_n consists of finite number of closed analytic Jordan curves. We denote by $\{G_n\}$ an exhaustion of G.

§2. $G-F_0SH$ function

We follow [1] for the definition and properties of Dirichlet functions. Let f be a continuous Dirichlet function on R with $f=0$ on $R-G$ and F be a regular closed snbset of G. Then there is a uniquely determined Dirichlet function f^F on R which minimizes the Dirichlet norm $\|g\|$ among Diri-