On the structure of the oriented cobordism ring modulo an equivalence

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§ 0. Introduction

Let I_n denote the subgroup of Ω_n^{so}, the oriented cobordism group of dim n, generated by all $[M'-M]\in\Omega_n^{so}$ such that M and M' have the same oriented homotopy type. Then $I_*=\sum_{n\geq 0} I_n$ is an ideal of Ω_*^{so}. In this paper we will determine the structure of Ω_*^{so}/I_* modulo 2-torsion.

THEOREM 0. 1. *The rank of* Ω_{4k}^{so}/I_{4k} *is one.*

THEOREM 0. 2. *For an odd prime* p, $\mathrm{Tor}\,(\Omega_*^{so}/I_*)\otimes \boldsymbol{Z}_p$ *is isomorphic to the polynomial ring* $\boldsymbol{Z}_p[\beta_{p-1},\cdots,\beta_{a\frac{p-1}{2}},\cdots]$, *where all* a *are positive integers so that* $a\left(\frac{p-1}{2}\right)$ *is not any form of* $\frac{p^j-1}{2}$ $(j=1,2,3,\cdots)$ *and the degree of* $\beta_{a\frac{p-1}{2}}$ *is* $2a(p-1)$.

Theorem 0. 1 has been proved in [4].

In § 1 we will show that there is an Ω_*^{so}-homomorphism d_*; $\Omega_*^{so}(F/0)\rightarrow\Omega_*^{so}$ so that $\mathrm{Cok}\,(d_*)$ is isomorphic modulo 2-torsion to $\mathrm{Tor}\,(\Omega_*^{so}/I_*)$. This homomorphism is originally found in [9, 10]. In § 2 we will compute $\mathrm{Cok}\,(d_*)$.

All manifolds will be compact, oriented and smooth.

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§ 1. Interpetation of Ω_n^{so}/I_n

Let M and M' be manifolds of dim n and a: $(M, \partial M)\rightarrow(M', \partial M')$, a homotopy equivalence of degree 1. (Both of $a|M$ and $a|\partial M$ are homotopy equivalences). We denote this by a *triple* (a, M, M'). If M and M' are closed, simply connected, or manifolds of dim n, then we call a triple (a, M, M') *closed*, *simply connected*, or *of dim n*. We define that closed triples of dim n (a, M, M') and (b, N, N') are *cobordant* if there exists a triple of dim $(n+1)$, (A, V, V') with $\partial V=M\cup(-N)$, $\partial V'=M'\cup(-N')$, $A|M=a$ and $A|N=b$. Then it is easily seen that this is an equivalence relation. As