On minimal points of Riemann surfaces, II.

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Dedicated to Prof. Yukinari Tôki on his 60th birthday

This paper is the continuation of the paper with the same title [1]. Definitions and terminologies in the previous paper will be used here also. Let R be a Riemann surface with positive boundary and let G be a domain in R. We suppose Martin's topologies M and M' are defined over $R+\Delta(R, M)$ and $G+\Delta(G,M')$, where $\Delta(R, M)$ and $\Delta(G, M')$ are sets of all Martin's boundary points of R and G respectively. Let $\underset{1}{\Delta}(R, M)$ (resp. $\underset{1}{\Delta}(G, M')$) be the set of all minimal points of $\Delta(R, M)$ (resp. $\Delta(G, M')$). Let $G(z,p)$ and $G'(z,p)$ be Green's functions of R and G respectively and let p^* be a fixed point in G. Put $G_\delta=\left\{z\in G: \dfrac{G'(z,p^*)}{G(z,p^*)}>\delta\right\}$. Then

THEOREM 1. (M. Brelot) [2]. *Let p be a point on ∂G. If p is irregular for the Dirichlet problem in G, the set of points in $\Delta(G, M')$ lying on p consists of only one point which is minimal.*

THEOREM 2. (M. Brelot) [3]. *Let $p\in\underset{1}{\Delta}(R, M)$. Then there exists a path Γ in R M-tending to p.*

THEOREM 3. (L. Naïm) [4]. *Let $\{p_i\}$ be a sequence in G_δ: $\delta>0$ such that $p_i\overset{M}{\longrightarrow}p\in\underset{1}{\Delta}(R, M)$. Then $\{p_i\}$ M'-tends to a point $q\in\underset{1}{\Delta}(G, M')$.*

We shall consider extensions of the above theorems. In this paper we use I and E operations. Let A and B be two hyperbolic domains in R such that $A\subset B$. Let $U(z)$ be a positive harmonic function in B. We denote by $\overset{B}{\underset{A}{I}}[U(z)]$ the upper envelope of continuous subharmonic functions in A smaller than $U(z)$ and vanishing on ∂A except a set of capacity zero. Let $V(z)$ be a positive harmonic function in A vanishing on ∂A except a set of capacity zero. We denote by $\overset{B}{\underset{A}{E}}[V(z)]$ the lower envelope of continuous superharmonic functions larger than $V(z)$. Then I and E have following properties:

1). *E and I are positive linear operators.*

2). *$I\,E[V(z)]=V(z)$.*

3). *If $U(z)$ is minimal in G and $I[U(z)]>0$, $EI[U(z)]=U(z)$.*