A note on the subdegrees of finite permutation groups*)

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Let G be a transitive permutation group on a finite set Ω. Let G_a be the stabilizer of $a\in\Omega$ in G. Let $\Delta_1=\{a\}$, $\Delta_2, \cdots, \Delta_r$ be the orbits of G_a on Ω (these are called the suborbits of (G, Ω)). Then we say that the permutation group (G, Ω) is of rank r, and we call $|\Delta_i|$'s the subdegrees of (G, Ω) (From the transitivity of (G, Ω), the $|\Delta_i|$'s are independent of the choice of $a\in\Omega$). When (G, Ω) is given, it is sometimes required to obtain the subdegrees. The purpose of this short note is to give a *practical* method to calculate the subdegrees when the structure of the group G is fairly known.

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NOTATION: Let G be a transitive permutation group on a set Ω, and let $H=G_a$ be the stabilizer of $a\in\Omega$. Let $\mathfrak{H}_1, \mathfrak{H}_2, \cdots, \mathfrak{H}_t$ be the sets of all H-conjugate subgroups of H (i.e., any subgroup $X\leqq H$ is contained in some and only one $\mathfrak{H}_i$). Moreover we fix an element $H_i\in\mathfrak{H}_i$ $(i=1, 2, \cdots, t)$. Let us define a partial order among $\mathfrak{H}_i$'s by $\mathfrak{H}_i\leqq\mathfrak{H}_j$ if there exist subgroups $X_i\in\mathfrak{H}_i$ and $X_j\in\mathfrak{H}_j$ such that $X_i\leqq X_j$. If $X_i\lneqq X_j$, we denote by $\mathfrak{H}_i<\mathfrak{H}_j$. Let us set $\Omega_i=H_i\backslash H$ (the right cosets of H by H_i), then H acts on Ω_i naturally. Let us set

$$I_\Omega(H_i)=\{b\in\Omega \mid b^h=b \text{ for any } h\in H_i\}, \text{ and}$$
$$I_{\Omega_j}(H_i)=\{b\in\Omega_j \mid b^h=b \text{ for any } h\in H_i\}.$$

(Note that the cardinality of these sets are independent of the choice of H_i in $\mathfrak{H}_i$ and of the choice of H_j in $\mathfrak{H}_j$.) Moreover let us set

$$A_{G,H}(H_i)=\{X\leqq H \mid \text{there exists } g\in G \text{ such that } X^g=H_i\}$$
$$(\text{where } X^g=g^{-1}Xg), \text{ and}$$

$$A_{H,H}(H_i)=\mathfrak{H}_i.$$

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