On the decompositions of function algebras

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Introduction. We shall be concerned with the decompositions of function algebras which are finer than the maximal antisymmetric decomposition. This fact was pointed out by Arenson [1] and Nishizawa [7], who respectively used the methods of Glicksberg [4] and Bishop [2]. Throughout the paper, underlying space X is a compact Hausdorff space and $C(X)$ denotes the algebra of all continuous complex-valued functions on X. We aim at the more systematic investigations of such decompositions of closed subspaces of $C(X)$ and of function algebras on X. Now we state our results in more detail, and define some usual notations which are used in this paper.

In §1, we consider a closed subspace B of $C(X)$. We show that the decompositions by the Glicksberg-Arenson method are always the decompositions by the Bishop-Nishizawa method, and that there exists the finest decomposition for each of the two methods. In §2, we consider a function algebra A on X. We show that there exists a one-to-one correspondence between p-sets in the base space X and p-sets in the maximal ideal space $\mathscr{M}(A)$. In virtue of this correspondence, we investigate the relations between the decompositions on X and those on $\mathscr{M}(A)$. In §3, we consider the rational function algebra $R(X)$ on a compact plane set X. In §4, we show that the difference between the maximal antisymmetric decomposition and the finer decomposition is of topological character. In §5, we shall construct three examples. Especially, Example 1 indicates that there must exists a decomposition which consists of more elementary components instead of the maximal antisymmetric components: Nevertheless, elementary components will not make simple algebras in general treatments.

Notations. $M(X)$ denotes the usual Banach space of all complex finite regular Borel measures on X. For μ in $M(X)$, we shall employ the notational abuse: $\mu(f)=\int f d\mu$. Let B be a closed subspace of $C(X)$, and we denote by $B^{\perp}$, $b(B^{\perp})$, and $b(B^{\perp})^{e}$ the total of anihilating measures of B, the unit ball of $B^{\perp}$, and the total of extreme points of $b(B^{\perp})$, respectively. Let E be a closed subset of X, and we denote by $f|E$ the restriction of the function f to E and $B|E=\{f|E: f\in B\}$. Let B_E denote the uniform closure of B/E in $C(E)$, and μ_E the restriction of μ to E: $\mu_E(K)=\mu(K\cap E)$. $M(E)$ can be considered as the closed subspace of $M(X)$ as the usual way.