Kählerian manifolds with vanishing Bochner curvature tensor satisfying $R(X, Y)\cdot R_1=0$

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1. Introduction. Let (M, J, g) be a Kählerian manifold of complex dimension n with the almost complex structure J and the Kählerian metric g.

The Bochner curvature tensor B of M is defined as follows:

$$B(X, Y)=R(X, Y)-\frac{1}{2n+4}[R^1X\wedge Y+X\wedge R^1Y+R^1JX\wedge JY$$
$$+JX\wedge R^1JY-2g(JX, R^1Y)J-2g(JX, Y)R^1\circ J]$$
$$+\frac{\text{trace } R^1}{(2n+4)(2n+2)}[X\wedge Y+JX\wedge JY-2g(JX, Y)J]$$

for any tangent vectors X and Y, where R and R^1 are the Riemannian curvature tensor of M and a field of symmetric endomorphism which corresponds to the Ricci tensor R_1 of M, that is, $g(R^1X, Y)=R_1(X, Y)$, respectively. $X\wedge Y$ denotes the endomorphism which maps Z upon $g(Y, Z)X-g(X, Z)Y$.

The tensor B has the properties similar to those of Weyl's conformal curvature tensor of a Riemannian manifold. For example, we can classify the restricted homogeneous holonomy groups of Kählerian manifolds with vanishing B, which seems to be an analogy of Kurita's theorem for the holonomy groups of conformally flat Riemannian manifolds [3], [5].

On the other hand, K. Sekigawa and one of the authors of present paper [4] classified conformally flat manifolds satisfying the condition

$$(*)\qquad R(X, Y)\cdot R_1=0 \qquad \text{for any tangent vectors } X \text{ and } Y,$$

where the endomorphism $R(X, Y)$ operates on R_1 as a derivation of the tensor algebra at each point of M.

In this paper, we shall prove

THEOREM. Let (M, J, g) be a connected Kählerian manifold of complex dimension n $(n\geqq 2)$ with vanishing Bochner curvature tensor satisfying the condition $(*)$, Then M is one of the following manifolds;

(I) A space of constant holomorphic sectional curvature.

(II) A locally product manifold of a space of constant holomorphic