Iterated mixed problems for d'Alembertians

By Rentaro AGEMI

§ 1. Introduction and main results

Let $\boldsymbol{R}_+^{n+1}$ be the open half space $\{(t, x); x=(x', x_n)=(x_1, \cdots, x_{n-1}, x_n), x_n>0\}$ with boundary $x_n=0$. By $(P, B_j; j=1, \cdots, l)$, briefly (P, B_j) we shall mean a mixed, or hyperbolic boundary value problem for a t-strictly hyperbolic operator P and boundary differential operators B_j:

$$P(t, x; D_t, D_x)u(t, x)=f(t, x) \quad \text{in } \boldsymbol{R}_+^{n+1},$$
$$B_j(t, x'; D_t, D_x)u(t, x', 0)=g_j(t, x') \quad (j=1, \cdots, l) \quad \text{on } \boldsymbol{R}^n.$$

Here $D_t=-i\dfrac{\partial}{\partial t}(i=\sqrt{-1})$, $D_k=-i\dfrac{\partial}{\partial x_k}$ and $D_x=(D_1, \cdots, D_n)$. Throughout this paper we assume that all the coefficients of P and B_j are C^∞ and constant outside a compact subset of $\boldsymbol{R}^{n+1}$. Moreover, Q^0 denotes the principal part of a differential operator Q and (τ, σ, λ) denote the dual variables of (t, x', x_n) respectively.

Let P_j^0 $(j=1, \cdots, m)$ be d'Alembertians:

$$P_j^0(t, x; \tau, \sigma, \lambda)=-\tau^2+a_j(t, x)^2\left(\lambda^2+\sum_{k=1}^{n-1}\sigma_k^2\right),$$
$$0<a_m(t, x)<\cdots<a_1(t, x)$$

and let B_j $(j=1, \cdots, m)$ be boundary differential operators of first order:

$$B_j^0(t, x'; \tau, \sigma, \lambda)=\lambda-\sum_{k=1}^{n-1} b_{jk}(t, x')\sigma-c_j(t, x')\tau,$$

where it will be assumed, unless otherwise indicated, that the $b_{jk}(t, x')$, $c_j(t, x')$ are real valued. Then for a permutation $\chi=\begin{pmatrix}1, & \cdots, & m\\ j_1, & \cdots, & j_m\end{pmatrix}$ a mixed problem $(P, {}^\chi B_j)=(P, {}^\chi B_j; j=1, \cdots, m)$ is said to be an iterated mixed, or boundary value problem, if the symbols of P^0 and ${}^\chi B_j^0$ have the following forms:

$$P^0(t, x; \tau, \sigma, \lambda)=\prod_{j=1}^{m} P_j^0(t, x; \tau, \sigma, \lambda),$$
$${}^\chi B_1^0(t, x'; \tau, \sigma, \lambda)=B_{j_1}^0(t, x'; \tau, \sigma, \lambda),$$
$${}^\chi B_k^0(t, x'; \tau, \sigma, \lambda)=B_{j_k}^0(t, x'; \tau, \sigma, \lambda)\prod_{h=1}^{k-1} P_{j_h}^0(t, x; \tau, \sigma, \lambda), \qquad (k=2, \cdots, m).$$