Kaehlerian manifolds with constant scalar curvature whose Bochner curvature tensor vanishes

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§ 1. Introduction

Let M be a Riemannian manifold of dimension $n \geqq 3$ and of class C^∞. We cover M by a system of coordinate neighborhoods $\{U; x^h\}$, where here and in the sequel the indices $h, i, j, k, \cdots$ run over the range $\{1, 2, \cdots, n\}$, and denote by $g_{ji}, \nabla_i, K_{kji}{}^h, K_{ji}$ and K the positive definite metric tensor, the operator of covariant differentiation with respect to the Levi-Civita of M connection, the curvature tensor, the Ricci tensor and the scalar curvature respectively.

A conformally flat Riemannian manifold is characterized by the vanishing of the Weyl conformal curvature tensor

$$C_{kji}{}^h = K_{kji}{}^h + \delta_k^h C_{ji} - \delta_j^h C_{ki} + C_k{}^h g_{ji} - C_j{}^h g_{ki}$$

and the tensor

$$C_{kji} = \nabla_k C_{ji} - \nabla_j C_{ki},$$

where

$$C_{ji} = -\frac{1}{n-2} K_{ji} + \frac{1}{2(n-1)(n-2)} K g_{ji},$$
$$C_k{}^h = C_{kt} g^{th}.$$

Ryan [4] proved

THEOREM *Let M be a compact conformally flat Riemannian manifold with constant scalar curvature. If the Ricci tensor is positive semi-definite, then the simply connected Riemannian covering of M is one of*

$$S^n(c),\ R \times S^{n-1}(c) \text{ or } E^n,$$

the real space forms of curvature c being denoted by $S^n(c)$ or E^n depending on whether c is positive or zero. (See also Aubin [1], Goldberg [3], Tani [6]).

He first proves that, in a conformally flat Riemannian manifold with constant scalar curvature K, we have