Finding a homeomorphism between almost homeomorphic manifolds

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§ 1. Introduction

Throughout this paper we shall only be concerned with the piecewise linear category of polyhedra and piecewise linear maps. In this paper we investigate the following problem; Let W_1 and W_2 be two PL manifolds whose interiors and boundaries are PL homeomorphic each other. When are W_1 and W_2 PL homeomorphic?

We obtain the result that such homeomorphism problem is closely related to the h-cobordism near the boundary (see THEOREM 2).

∂M and $Int\ M$ stand for the boundary and the interior of the manifold M. $\cong$ means PL homeomorphic. $I=[0, 1]$ is a closed unit interval. $\#X$ means the order of a set X.

§ 2.

DEFINITION 1. Let W_i $(i=1, 2)$ be bounded manifolds. When $\partial W_1 \cong \partial W_2$ and $Int\ W_1 \cong Int\ W_2$, we say W_1 is *almost homeomorphic* to W_2. And we define $\mathscr{A}(W)=$ set of PL homeomorphism classes of PL manifolds which are almost homeomorphic to W.

PROPOSITION 1. ([2. *Th.* 2, 4]) *Let* W_j^n $(j=1, 2)$ *be compact bounded* n*-manifolds* $(n \geqq 6)$. *Then* $Int\ W_1^n \cong Int\ W_2^n$ *if and only if* W_1 *and* W_2 *are boundary h-cobordant i.e. there are h-cobordisms* $(U^{(i)};\ \partial W_2^{(i)},\ M^{(i)})$ *such that*

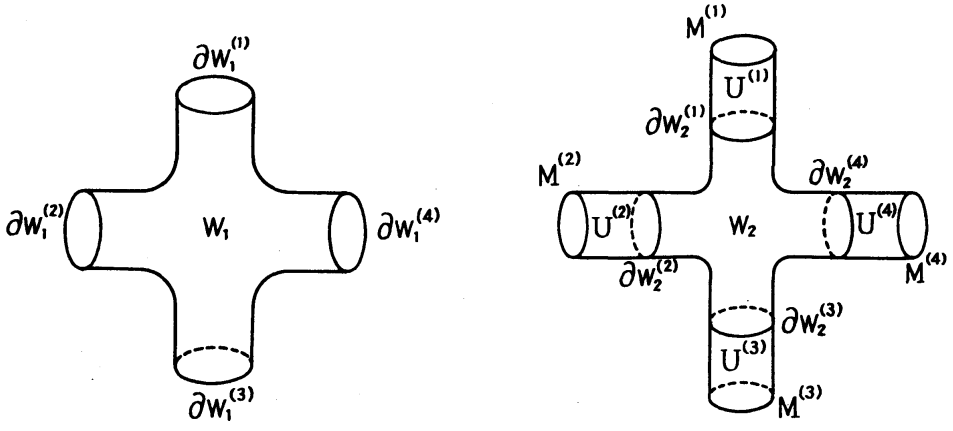

Fig. 1.