A characterization of A_7 and M_{11}, I

Dedicated to Professor Yataro Matsushima on his 60th birthday

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1. Introduction

In this paper we shall prove the following theorem.

THEOREM. *Let G be a doubly transitive group on the set $\Omega=\{1,2,\cdots,n\}$ containing no regular normal subgroup. If the stabilizer $G_{1,2}$ of points 1 and 2 is isomosphic to a simple group $PSL(2,2^m)$, then one of the following holds:*

(1) $n=7$ *and G is the alternating group A_7 of degree seven,*

(2) $n=12$ *and G is the Mathieu group M_{11} of degree eleven.*

In [12] Yamaki proved Theorem in the case $m=2$. Therefore we may assume $m>2$. A proof of Theorem is similar to that of [7].

Let X be a subset of a permutation group. Let $F(X)$ denote the set of all fixed points of X and $\alpha(X)$ be the number of points in $F(X)$. $N_G(X)$ acts on $F(X)$. Let $\chi_1(X)$ and $\chi(X)$ be the kernel of this representation and its image, respectively. The other notation is standard.

2. Preliminaries

Let $G_{1,2}$ be $PSL(2,2^m)$ with $m>2$. Let K be a Sylow 2-subgroup of $G_{1,2}$. Then $N_{G_{1,2}}(K)$ is a complete Frobenius group with complement H. Let I be an involution of G with the cycle structure $(1,2)\cdots$. Then I normalizes $G_{1,2}$.

LEMMA 1. *It may be assumed that the action of I on $G_{1,2}$ is trivial or the field automorphism.*

PROOF. Let ϕ be a homomorphism of $<I,G_{1,2}>$ into Aut $PSL(2,2^m)$. If ker $\phi\neq 1$ and $\phi(I)\neq 1$, then we can replace I by an element $(\neq 1)$ of ker ϕ. If ker $\phi=1$, then I induces an outer automorphism. Since $<I,G_{1,2}>$ has two classes of involution, I is conjugate to the field automorphism.

By Lemma 1 I is contained in $N_G(H)\cap N_G(K)$. Let τ be an involution of $C_K(I)$. Let τ fix i points of Ω, say $1,2,\cdots,i$. By a theorem of Witt [11, Th. 9.4] $\chi(\tau)$ is doubly transitive on $F(\tau)$.

LEMMA 2. $n=i(\beta i-\beta+\gamma)/\gamma$, *where β is the number of involutions with*