On a certain subspace of the Riemannian projective recurrent space

By Toshikiyo YAMADA

§ 0. Introduction

Riemannian spaces which admit some recurrent tensors have been studied by many authors. Recently, T. Miyazawa and Gorō Chūman [1] have studied the subspaces of a Riemannian recurrent space. In this paper, we would like to further study the subspaces of the Riemannian projective recurrent spaces.

The Riemannian space V_m may be called a projective recurrent space if Weyl's projective curvature tensor

$$P_{kji}{}^{h} = \bar{R}_{kji}{}^{h} - \frac{1}{m-1}(\bar{R}_{ji}\delta_k^h - \bar{R}_{ki}\delta_j^h) \tag{0.1}$$

satisfies the relation

$$\nabla_l P_{kji}{}^{h} = K_l P_{kji}{}^{h}, \tag{0.2}$$

where ∇_l denotes a covariant differentiation with respect to the metric tensor g_{ij} of the V_m. We will call K_l in (0.2) the vector of recurrence of the space.

The present author wishes to express here his sincere thanks to Professor Yoshie Katsurada and Doctor Tamao Nagai for their kindly guidance and encouragement.

§ 1. Preliminary

Let us consider an n-dimensional subspace V_n, of local coordinate y^a, immersed in an m-dimensional Riemannian space V_m of local coordinate x^i. Let $B_a^i = \partial x^i/\partial y_a$, then the rank of the matrix (B_a^i) is n, where the indices $h, i, j, \cdots$, take the values $1, \cdots, m$ and the indices $a, b, c. \cdots$, the values $1, \cdots, n\,(m>n)$. We have the components g_{ab} of the fundamental tensor for V_n given by the relation $g_{ab} = B_a^i B_b^j g_{ij}$, g_{ij} being the components of the fundamental tensor for V_m.

Let $N_P (P=n+1, \cdots, m)$ be unit normals to the V_m and mutually orthogonal, then we have the relations

$$g_{ij} N_P^i N_P^i = e_P, \quad g_{ij} N_P^i N_Q^j = 0\,(P \neq Q), \quad g_{ij} B_a^i N_P^j = 0, \tag{1.1}$$