Examples of the manifolds $f^{-1}(0)\cap S^{2n+1}$, $f(Z)=Z_0^{a_0}+Z_1^{a_1}+\cdots+Z_n^{a_n}$

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Consider the polynomials $f(z)=Z_0^{a_0}+Z_1^{a_1}+\cdots+Z_n^{a_n}$, $a_i\geqq 2$, $z_i\in \boldsymbol{C}\,(i=0,1,2,\cdots,n)$ and closed differentiable manifolds of dim $(2n-1)$, $K_a=f^{-1}(0)\cap S^{2n+1}$, where S^{2n+1} denotes the unit sphere in $\boldsymbol{C}^{n+1}$. The purpose of this paper is to give examples which shows what manifolds K_a are when $(a_0,a_1,\cdots,a_n)=(2,2,\cdots,2,p,q)$, $q\equiv 0(p)$ and $n\geqq 3$. This paper is a continuation of [1], so we will use the same notations as them in [1]. Let $q\equiv 0(p)$ be satisfied. Then $K_{a'}$, $a'=(2,2,\cdots,2,p,q-1)$ is a homotopy sphere which is denoted by Σ in the sequel if and only if n is odd or both p and $q-1$ are odd in case of n being even. This is an easy consequence of [3, §14]. In the sequel we assume that a and a' are as stated above. Unless otherwise stated, a manifold means a smooth manifold.

THEOREM 1. *Let $n\geqq 3$ and $q\equiv 0(p)$.*

(i) *If n is odd, then K_a is diffeomorphic to $(S^{n-1}\times S^n)_1\#(S^{n-1}\times S^n)_2\#\cdots\#(S^{n-1}\times S^n)_{p-1}\#\Sigma$ when p is odd or both p and q/p are even, and to $\partial D(\tau_{S^n})_1\#\cdots\#\partial D(\tau_{S^n})_{p/2}\#(S^{n-1}\times S^n)_{p/2+1}\#\cdots\#(S^{n-1}\times S^n)_{p-1}\#\Sigma$ when p is even and q/p is odd.*

(ii) *If n is even, $p=3$, and $q\equiv 0(6)$, then K_a is diffeomorphic to $(S^{n-1}\times S^n)\#(S^{n-1}\times S^n)\#\Sigma$.*

At first we consider ths case when n is odd. Let F_a be a fiber of Milnor fibering associated to the polynomial f and $\bar{F}_a$ the closure of F_a in S^{2n+1} [5]. Now we recall the exact esquence $0\to H_n(K_a)\to H_n(\bar{F}_a)\xrightarrow{\Psi} H_n(\bar{F}_a, K_a)\xrightarrow{\partial} H_{n-1}(K_a)\to 0$. [5]

To know the modules $H_n(K_a)$ and $H_{n-1}(K_a)$ we must examine the matrix

$$\Psi=\begin{pmatrix} A-{}^tA, & A, & \cdots & A \\ -{}^tA, & A-{}^tA, & \ddots & \vdots \\ \vdots & \ddots & \ddots & A \\ -{}^tA, & \cdots & -{}^tA, & A-{}^tA \end{pmatrix}$$