On Maslov's canonical operator

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Introduction

For a global treatment of quasi-classical approximation of the Schrödinger equation, V. P. Maslov [16] introduced and discussed quite systematically the notions of Lagrangean manifolds and associated canonical operators on them. The underlying ideas are known since long and their applications to the study of partial differential equations are not quite new. (e.g., Lax [11], Lewis [13], Ludwig [14]). Even very close considerations to Maslov's are not so rare, as are found in works of Keller [9], [10], Ludwig [15] and others, and Hörmander's recent notion of Fourier integral operators [7] is in a sense one of them, though apparently different. Maslov's original exposition [16], however interesting and stimulating its content be, seems to be not necessarily well arranged and even to contain certain unclearness, thus letting the reader sometimes difficult to grasp its validity. As to this J. Leray [12] gave a review but without any remark on the connections of Maslov's canonical operator and Hörmander's Fourier integral operator. However, we believe that these two notions are deeply concerned and in a sense variants of the same thing, and thus are not quite satisfied with this situation. So we describe below what Maslov's canonical operators should be. Our exposition will be thus quite close to Hörmander's Fourier integral operator. In fact, when I had completed my first draft I was then informed about Duistermaat [3]. The interpretation of Maslov's canonical operator by him and me are essentially the same. However, I choose a different symbol class (cf. Definition 2.1.1) to define canonical operators, and I personally believe that this choice of symbol class is an essential simplification from Maslov's original and with this I can smoothly apply Hörmander's method. On the other hand, Duistermaat [3] starts from a smaller symbol class and thus his discussion runs in a sense in the reversed order with respect to mine. Any way, I publish here only the definitions and elementary properties of the so-called canonical operators and omit their calculi, since their applications are done just in the same way as Fourier integral operators, that is, one needs only to construct canonical relations as an analogy to homogeneous canonical relations, and then to establish their calculi. Here, however, the degree of product symbols is