On structures of certain L^2-well-posed mixed problems for hyperbolic systems of first order

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§ 1. Introduction and results.

Let P be an x_0-strictly hyperbolic $2m\times 2m$-system of differential operators of first order defined over a C^∞-cylinder $\boldsymbol{R}^1\times\Omega\subset\boldsymbol{R}^{n+1}$. Let B be an $m\times 2m$-system of functions defined on the boundary Γ of $\boldsymbol{R}^1\times\Omega$. We consider the following mixed problem:

$$(P, B)\ \begin{cases} P(x, D)\, u(x)=f(x) & x\in\boldsymbol{R}^1\times\Omega \quad (x_0>0),\\ B(x)\, u(x)=g(x) & x\in\Gamma \quad (x_0>0),\\ u(x)=h(x) & x\in\boldsymbol{R}^1\times\Omega \quad (x_0=0),\end{cases}$$

where $x=(x_0, x_1, \cdots, x_n)$, $D_j=\dfrac{1}{\sqrt{-1}}\dfrac{\partial}{\partial x_j}=\dfrac{1}{i}\dfrac{\partial}{\partial x_j}$ and $D=(D_0, \cdots, D_n)$. We consider L^2-well-posedness of (P, B) in the follwing sense:

DEFINITION 1.1. *The problem (P, B) is said to be L^2-well-posed if there exist positive constants C, T such that for every $f\in H_1((-\infty, T)\times\Omega)$ with $f=0\,(x_0<0)$, $g=0$ and $h=0$, there exists a unique solution $u\in H_1((-\infty, T)\times\Omega)$ with $u=0\,(x_0<0)$ which satisfies the inequality*:

$$\int_0^T \|u\|^2_{0,\Omega}\,dt\leqq C\int_0^T \|f\|^2_{0,\Omega}\,dt,$$

where $H_k(G)$ is the Sobolev space with its norm $\|\cdot\|_{k,G}$.

The aim of the present article is, under somewhat strong but general restriction on the operator P, to describe, in terms of the cotangent space of $\boldsymbol{R}^1\times\partial\Omega=\Gamma$, the relations among the coefficients of boundary operator B for L^2-well-posed problem (P, B). These relations are useful for the investigation of the propagation of singularities of solutions for our problems. For example they determine whether there exist lateral waves or not. Applying the relations we prove the existance of the solution of our problem. But in contrast with the recent development of the Cauchy problems, we must essentially use the energy estimate, because of the existance of glancing rays with the associated non-vanishing reflection coefficients in these cases.