Quasi-invariant measures on linear topological spaces

By Yasuji TAKAHASHI

§ 1. Introduction

In [1], Dao-Xing has shown that the following:

THEOREM *Let H be a separable Hilbert space, and let $\mathfrak{F}$ be the totality of weak Borel sets in H. Let Φ be a linear subspace of H, and suppose that Φ itself is a complete σ-Hilbert space with respect to the sequence of inner products $(\varphi, \psi)_n$, $n=1, 2, 3, \cdots$*
where $(\varphi, \varphi)_1 \leqq (\varphi, \varphi)_2 \leqq \cdots$.
Also, suppose that the inclusion mapping T from Φ into H is continuous. For each n, let Φ_n denote the completion of Φ with respect to the inner product $(\varphi, \psi)_n$. Then, the following conditions are equivalent.

(1) *There exists a Φ-quasi-invariant finite measure (non-trivial) μ on $(H, \mathfrak{F})$.*

(2) *There exists n such that the inclusion mapping T can be extended to a Hilbert-Schmidt operator from Φ_n into H.*

In the Dao-Xing's original Theorem, it is necessary that μ is regular. In this paper, we shall show that this assumption can be taken off, furthermore this theorem can be extended to complete σ-normed spaces.

Throughout this paper (except for § 2.1°. and § 5.), we shall assume that linear spaces are with real coefficients.

§ 2. Basic definitions and well known results

1°. p-absolutely summing operators ($1 \leqq p < \infty$)

Let E and F be Banach spaces.

DEFINITION 2.1.1. *Let $\{x_n\}$ be a sequence from a Banach space E. $\{x_n\}$ is called scalarly l_p if for each continuous linear functional $x^* \in E^*$, we have the inequality*

$$\sum_{n=1}^{\infty} |x^*(x_n)|^p < \infty .$$

$\{x_n\}$ is called absolutely l_p if $\sum_{n=1}^{\infty} \|x_n\|^p < \infty$.

DEFINITION 2.1.2. *A linear operator T from E into F is called p-*