# Notes on Green lines

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## 1. Introduction

Let $R$ be a hyperbolic Riemann surface and $g(z)=g(z, z_0)$ be the Green function on $R$ with a fixed pole $z_0$ in $R$. For the following definitions and properties of Green lines and compactifications of $R$, we refer to Sario-Nakai [7] and Constantinescu-Cornea [2] respectively. We consider the Green lines issuing from the fixed point $z_0$. The set $L$ of all Green lines admits the Green measure $m$. A Green line $l$ for which $\inf_{z\in l} g(z)=0$ is called a regular Green line. Any regular Green line tends to the ideal boundary of $R$ as $g(z)\to 0$. The set of all regular Green lines is denoted by $L_r$. It is known (Brelot-Choquet [1]) that $m(L-L_r)=0$.

Let $R^*$ be a resolutive compactification of $R$ and $\mu_z$ be the harmonic measure on the ideal boundary $\Delta=R^*-R$ with respect to $z\in R$. We are interested in the behavior of $l\in L_r$ in $R^*$. We set $e(l)=\bar{l}-l\cup\{z_0\}$ with $\bar{l}$ the closure of $l$ in $R^*$. We call $e(l)$ the end part of $l$ in $R^*$. Given a subset $S\subset\Delta$ we write $\tilde{S}=\{l\in L_r | e(l)\cap S\neq\phi\}$ and $\check{S}=\{l\in L_r | e(l)\subset S\}$. Let $C(\Delta)$ be the set of all bounded continuous functions on $\Delta$. We set $C_D(\Delta)=\{f\in C(\Delta) | H_f^{R,R^*}\in HD(R)\}$. If $C_D(\Delta)$ is dense in $C(\Delta)$ with respect to the uniform convergence topology, then $R^*$ is said to be a regular compactification of $R$ (Maeda [4]).

In this paper we shall prove the following theorems:

THEOREM 1. *Let $R^*$ be a resolutive compactification of $R$. For every compact set $K$ (resp. open set $U$) in $\Delta$,*

$$\overline{m}(\check{K})\leq\mu_{z_0}(K), \quad \underline{m}(\tilde{U})\geq\mu_{z_0}(U),$$

*where $\overline{m}$ and $\underline{m}$ are the outer and inner measures induced by $m$. For every Baire set $S$ in $\Delta$, $\overline{m}(\check{S})\leq\mu_{z_0}(S)\leq\underline{m}(\tilde{S})$.*

COROLLARY 1. *Let $R^*$ be resolutive. If $R^*$ is metrizable, then for every Borel set $S$ in $\Delta$, $\overline{m}(\check{S})\leq\mu_{z_0}(S)\leq\underline{m}(\tilde{S})$.*

COROLLARY 2. (*i*) *Let $R^*$ be resolutive and $\Gamma$ be the harmonic boundary of $R^*$. If $R^*$ is metrizable, then $m(\tilde{\Gamma})=1$.*

(*ii*) *Let $R_M^*$ be the Martin compactification of $R$ and $\Delta_1$ be the set of all minimal points of $\Delta_M=R_M^*-R$. Then $m(\tilde{\Delta}_1)=1$.*