Analytic Functions on Some Hyperbolic Riemann Surfaces

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Introduction

Z. Kuramochi gave in his paper [4] a very interesting theorem, which can be stated as follows.

Theorem of Kuramochi. Let R be a hyperbolic Riemann surface of the class O_{HB} (resp. O_{HD}). Then, for any compact subset K of R such that $R-K$ is connected, $R-K$ as an open Riemann surface belongs to the class O_{AB} (resp. O_{AD}).

The theorem was proved by using the existence of points of positive harmonic measure on Martin or Kuramochi boundary. It is known that the existence of points of positive harmonic measure on the Martin or the Kuramochi boundary is equivalent to the existence of those points on the Wiener or the Royden boundary. Then there were questions whether there exists a hyperbolic Riemann surface, which has no boundary points with positive harmonic measure on the Royden or the Wiener boundary and has yet the same property as stated in the theorem of Kuramochi. To these questions N. Toda and K. Matsumoto [17] and K. Matsumoto [11] gave answers in the positive and proved that $R \in O_{A^0_X} \cap U_S$ implies $R-K \in O_{AX}$ ($X=B$ or D) for every compact subset K of R with connected complement.

In this paper we shall deal with a problem similar to the above by considering the class O_{AN} and capacities instead of harmonic measures on the Royden boundary. The main pourpose of this paper is to show a theorem similar to the above: $R \in O_{A^0_N} \cap U_N$ implies $R-K \in O_{AN}$ for any compact subset K of R with connected complement.

1. Capacity on the Royden boundary (cf. [14, 15])

Let R be a hyperbolic Riemann surface. For a subset A of R, we denote by ∂A the (relative) boundary of A in R. We call a closed or open subset A of R is regular if ∂A is non-empty and consists of at most a countable number of analytic arcs clustering nowhere in R. We fix a closed disk K_0 in R once for all and let $R_0=R-K_0$.

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