A characterization of A_7 and M_{11}, II

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1. Introduction

In this paper we shall prove the following theorem.

THEOREM 1. *Let G be a doubly transitive group on the set* $\Omega=\{1, 2, \cdots, n\}$. *If the stabilizer* $G_{1,2}$ *of points* 1 *and* 2 *is isomorphic to a simple group* $PSL(2, q)$, $q\equiv 3$ *or* 5 (*mod* 8), *then one of the following holds*:

(1) *G has a regular normal subgroup,*

(2) $n=7$ *and G is the alternating group* A_7 *of degree seven,*

(3) $n=12$ *and G is the Mathieu group* M_{11} *of degree eleven.*

In [17] Yamaki proved Theorem in the case $q=5$. Therefore we may assume $q\geq 11$.

Let X be a subset of a permutation group. Let $F(X)$ denote the set of all fixed points of X and $\alpha(X)$ be the number of points in $F(X)$. $N_G(X)$ acts on $F(X)$. Let $\chi_1(X)$ and $\chi(X)$ be the kernel of this representation and its image, respectively. The other notation is standard.

2. Preliminaries

Let us assume G has no regular normal subgroup. Let $G_{1,2}$ be $PSL(2, q)$, $q\equiv 3$ or 5 (8). Let $K=<\tau, \tau'>$ be a Sylow 2-subgroup of $G_{1,2}$. Let I be an involution of G with the cycle structure $(1, 2)\cdots$. Then I normalizes $G_{1,2}$ and hence we may assume I normalizes K and $[I, \tau]=1$. Let τ fix i points of Ω, say $1, 2, \cdots, i$. Every involution of G is conjugate to an involution in $IG_{1,2}$.

LEMMA 1. *It may be assumed that the action of I on* $G_{1,2}$ *is trivial or an outer automorphism.*

PROOF. Since $q-1\not\equiv 0$ (8), $[P\Gamma L(2, q): PGL(2, q)]$ is odd. Let ϕ be the homomorphism of $<I, G_{1,2}>$ into Aut $G_{1,2}$. If ker $\phi\neq 1$, we can replace I by an element $(\neq 1)$ of ker ϕ.

LEMMA 2. *If I does not centralize* $G_{1,2}$, *then G has just one class of involutions.*

PROOF. Since $<I, G_{1,2}> = PGL(2, q)$ has two classes of involutions, every involution in $IG_{1,2}$ is conjugate to I.