On the exact ranges of complex manifolds

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Introduction

In [3] Kobayashi gave the following interesting statement: let M be a complex manifold on which a complex lie group acts transitively, then M can not be hyperbolic. The method of the proof and the fact that the complex line C, which is a typical, non-taut, non-tight and non-hyperbolic space, plays an essential role in the proof suggested to us to consider the same circumstances from our point of view. Our aim is to show that the set of holomorphic mappings $H(C, M)$ can neither be normal nor equicontinuous. For this purpose we introduce the property (P) instead of "two-fold assigning" property. The property (P) is in a sense a localization of "two-fold assigning property". The property (P) was already introduced in [4] and made it possible to improve Wu's theorem, cf. Lemma 4.1, [4]. In this paper the property (P) and the notion of exact range, see Definition 2.3, which is laso a localization of the notion introduced in [4], will prove themselves effective for our purpose.

The purpose of § 3 is to make some remarks on Kobayashi's statement cited above.

§ 1. Preliminaries.

Through this paper complex manifolds are all assumed connected and second countable. For two complex manifolds M and N we denote by $H(M, N)$ the space of all holomorphic mappings of M to N. The space $H(M, N)$ can be topologized by so-called compact-open topology. Since by assumption M and N are second countable, $H(M, N)$ is also second countable, and the compactness of a subset of $H(M, N)$ is verified by its sequential compactness. By the same assumption the complex manifold N is metrizable and we can construct a distance function d_N on N which metrizes N, cf. Kelley [1]. So we can speak of the convergence of a sequence of $H(M, N)$ making use of the distance function d_N.

A sequence $\{f_i\} \subset H(M, N)$ converges *compact-uniformly* in M if and only if it converges uniformly on every compact subset of M. The compact-uniform limit of a sequence of $H(M, N)$ belongs to $H(M, N)$. A sequence $\{f_i\} \subset H(M, N)$ is said to be *compactly divergent* if and only if for