A characterization of A_7 and M_{11}, III

Dedicated to Professor Kiiti Morita on his 60th birthday

By Hiroshi KIMURA

1. Introduction

In this paper we shall prove the following theorem.

THEOREM 1. *Let G be a doubly transitive group on the set $\Omega=\{1, 2, \cdots, n\}$. If the stabilizer $G_{1,2}$ of points 1 and 2 is isomorphic to the Janko's simple group $J(11)$ of order $2^3\cdot 3\cdot 5\cdot 7\cdot 11\cdot 19$ or a group $R(q)$ of Ree type, then G has a regular normal subgroup.*

By Walter's theorem a simple group with abelian Sylow 2-subgroups is isomorphic to $J(11)$, $R(q)\,(q\neq 3)$, $PSL(2, 2^m)$ or $PLS(2, q)$ with $q\equiv 3$ or 5 (mod 8). Theorefore by Theorem 1 and theorems in [7] we have the following.

THEOREM 2. *Let G be a doubly transitive group on the set $\Omega=\{1, 2, \cdots, n\}$. If $G_{1,2}$ is isomorphic to a simple group with abelian Sylow 2-subgroups, then G is isomorphic to the alternating group A_7 of degree seven, the Mathieu group M_{11} of degree eleven or G has a regular normal subgroup.*

Let X be a subset of a permutation group. Let $F(X)$ denote the set of all fixed points of X and $\alpha(X)$ be the number of points in $F(X)$. $N_G(X)$ acts on $F(X)$.

Let $\chi_1(X)$ and $\chi(X)$ be the kernel of this representation and its image, respectively. The other notation is standard.

2. Preliminaries

Let G be a doubly transitive group on Ω not containing a regular normal subgroup such that $G_{1,2}$ is isomorphic to $J(11)$ or $R(q)$. Let K be a Sylow 2-subgroup of $G_{1,2}$. Then K is an elementary abelian 2-group of order 8. Let I be an involution of G with the cycle structure $(1, 2)\cdots$. Then I normalizes $G_{1,2}$. Since $\mathrm{Aut}\,(G_{1,2})/\mathrm{Inn}\,(G_{1,2})$ is of odd order, we may assume I centralizes $G_{1,2}$. Let τ be an involution of K. Let τ fix i points of Ω, say $1, 2, \cdots, i$. Since every involution of G is conjugate to an involution in $IG_{1,2}$, it is conjugate to I or $I\tau$.

Let d be the number of elements in $G_{1,2}$ inverted by I. Set $\gamma=[G_{1,2} : C_G(\tau)\cap G_{1,2}]$. Let β be the number of involutions with the cycle structures