On a PL embedded 2-sphere in 4-manifold

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§ 0.

Let M^{2n} be a simply connected differentiable manifold, and let $\xi \in \pi_n(M^{2n})$ be a given homotopy class of maps $S^n \to M^{2n}$. It is known that if $n>2$, the class ξ can be represented by a differentiable imbedding $f\colon S^n \to M^{2n}$. This follows from a reasoning similar to the one used by H. Whitney to prove that every differentiable n-manifold can be differentiably imbedded in Euclidean $2n$ space. For $n=1$, let $F_{p,q}$ be a compact connected orientable surface of genus p with q boundary components, where q may be equal to 0. Let $a_1, \cdots, a_p, b_1, \cdots, b_p, C_1, \cdots, C_{q-1}$ be standard generators for the $H(F_{p,q}: Z)$. Here the C_i correspond to consistently oriented boundary circles (one is omitted because it is homologous to the sum of the others), and the a_i and b_i are standard curves on $F_{p,q}$, chosen so that $a_i \cap a_j = b_i \cap b_j = a_i \cap b_j = \phi$ if $i \neq j$ and a_i, b_i intersect nicely at one point. Then S. Suzuki [5] proved the following:

SUZUKI'S THEOREM. *A non zero homology class* $\sum_{i=1}^{p} \alpha_i a_i + \sum_{i=1}^{p} \beta_i b_i + \sum_{i=1}^{q-1} \gamma_i C_i$ *of* $H_1(F_{p,q}: Z)$ *is representable by a simple closed curve on* $F_{p,q}$ *if and only if one of the following two conditions is satisfied*:

(1) *Not all the* α_i *and* β_i *are zero and the g.c.d*$(\alpha_1, \cdots, \alpha_p, \beta_1, \cdots, \beta_p)=1$,

(2) $\alpha_i = \beta_i = 0$ *for* $1 \leqq i \leqq p$ *and* $|\gamma_i| \leqq 1$ *for* $1 \leqq i \leqq q-1$ *and all non zero* γ_i *have the same sign.*

For $n=2$, whether or not $\xi \in H_2(M^4)$ is representable by a differentiable imbedding $f\colon S^2 \to M^4$ depends on the class ξ ([2], [6]). On the other hands there exists no example whose class $\xi \in H_2(M^4)$ is not representable by a PL imbedding $f\colon S^2 \to M^4$. So we attack a problem representing a class $\xi \in H_2(M^4)$ by a PL embedding $f\colon S^2 \to M^4$. And we obtain a following.

THEOREM. *Let* M^4 *be a* 1*-connected closed PL* 4*-manifold. Then any* 2*-dim. homology class* $\xi \in H_2(M \# k(S^2 \times S^2))$ *is representable by a PL embedding* $f\colon S^2 \to M \# k(S^2 \times S^2)$ *for some* $k \geqq 0$ *where* $M \# k(S^2 \times S^2)$ *is a connected sum of M with k-copies of* $S^2 \times S^2$.