On surfaces in 3-sphere: Prime decompositions

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0. Introduction

Throughout this paper, we work in the piecewise linear category, consisting of simplicial complexes and piecewise linear maps. The theorems concern "knot types" of a connected, closed (=compact, without boundary), oriented surface (=2-dimensional manifold) F in the 3-dimensional sphere S^3 with a fixed orientation.

In the previous paper [25], we showed a unique prime decomposition theorem for special linear graphs in S^3 as generalization of knots [23] and links [12], see [20] and also [2], [10], [26], [27]. In the paper, we shall formulate a prime decomposition theorem for pairs $(F\subset S^3)$'s as the same way as that of [25] and [27] except for obvious modifications, and discuss the uniqueness of the prime decompositions.

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1. Prime Decompositions for $(F\subset S^3)$

In the paper, homeomorphism and isomorphism are denoted by the same symbol $\cong$, while $\approx$, $\simeq$ and $\sim$ refer, respectively, to isotopy, homotopy and homology. ∂X, $cl(X)$ and $°X$ denote, respectively, the boundary, the closure and the interior of a manifold X, and when applied to oriented objects these respect orientations. By $\boldsymbol{Z}$ we shall denote the infinite cyclic group.

We shall say that a submanifold X of a manifold Y is *properly embedded* (or simply *proper*) if $X\cap\partial Y=\partial X$.

By D^n and S^{n-1} we shall denote the standard n-cell and the standard $(n-1)$-sphere ∂D^n, respectively. We always assume that S^3 has the right-handed orientation.

For a connected surface F, $g(F)$ stands for the genus of F.

We shall now formulate the prime decomposition for pairs $(F\subset S^3)$ of closed, connected and oriented surfaces in S^3.

1.1. Definition. Two pairs $(F_1\subset S^3)$ and $(F_2\subset S^3)$ are said to be *congruent*, denoted by $(F_1\subset S^3)\cong(F_2\subset S^3)$, if there is an orientation-preserving