# Characteristic Classes of Foliated Principal $GL_r$-Bundles

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## Introduction

Let $M$ be a paracompact Hausdorff differentiable ($C^\infty$) manifold of dimension $n$ and $\mathscr{F}$ a differentiable ($C^\infty$) condimension $q$ foliation on $M$. We denote the manifold $M$ with the foliation $\mathscr{F}$, by $(M, \mathscr{F})$. Let $GL_r = GL(r, R)$ denote the group of $r \times r$ non-singular matrices over real numbers. A *foliated* principal $GL_r$-bundle $E(M, p, GL_r)$ over the $(M, \mathscr{F})$ is a differentiable ($C^\infty$) principal $GL_r$-bundle $p: E \rightarrow M$, such that $E$ has a right $GL_r$-invariant differentiable ($C^\infty$) foliation $\mathscr{F}_E$, where each leaf is a covering of a leaf of $\mathscr{F}$. (Cf. P. Molino [4].) $\mathscr{F}_E$ is called a *lifted foliation* of $\mathscr{F}$.

We generalize the Bott's construction of characteristic classes of a foliation (cf. R. Bott [1] and P. Molino [6]) to the foliated principal $GL_r$-bundles and we obtain several vanishing theorems of the characteristic classes. In particular, these theorems are remarkable in the case where $E(M, p, GL_r)$ admits a transverse projectable connection. P. Molino [6] obtains these theorems for the frame bundle of the normal bundle of the foliation $(M, \mathscr{F})$. However, if $M$ has two foliations $\mathscr{F}$ and $\mathscr{F}'$ of condimensions $q$ and $r$ respectively ($q \geqq r$) such that the tangent subbundle $F$ of $\mathscr{F}$ is a subbundle of the tangent subbundle $F'$ of $\mathscr{F}'$, then we can construct a foliated principal $GL_r$-bundle $E(M, p, GL_r)$ over $(M, \mathscr{F})$ and our generalized arguments of characteristic classes are applied to such foliated principal bundles.

Some applications of our theorems will be given in a subsequent note.

## § 1. A transverse connection

A connection on the foliated principal $GL_r$-bundle $E(M, p, GL_r)$ over the foliated manifold $(M, \mathscr{F})$ is said to be a *transverse* connection, if leaves of the lifted foliation $\mathscr{F}_E$ of $\mathscr{F}$ are horizontal for the connnection. (Cf. P. Molino [4].) In this section, we shall introduce characteristic classes of the foliated principal $GL_r$-bundle by the notion of the transverse connection.

Let $gl_r$ denote the Lie algebra of $GL_r$ and $I(gl_r)$ denote the algebra of invariant polynomials of $gl_r$. Let $\nabla^1$ be a transverse connection on the $E(M, p, GL_r)$. It is easy to see that the $E(M, p, GL_r)$ admits a transverse connection.