Analytic functions in a neighbourhood of irregular boundary points

By Zenjiro KURAMOCHI

(Received April 30, 1975)

The present paper is a continuation of the previous paper with title "Analytic functions in a lacunary end of a Riemann surface"[1]. We use the same notions and terminologies in the previous one. Let G be an end of a Riemann surface $\in O_g$ (we denote by O_G the class of Riemann surfaces with null boundary) and $G'=G-F$ be a lacunary end and let $p\in\Delta_1(M)$ be a minimal boundary point relative to Martin's topology M over G with irregularity $\delta(p)=\overline{\lim_{\substack{M\\ z\to p}}}\, G(z,p_0)>0$, where $G(z,p_0)$: $p_0\in G'$ is a Green's function of G'. Then Theorems 2, 3 and 4 in the previous show that analytic functions in G' of some classes have similar behaviour at p as p is an inner point of G'. We shall show these theoremes are valid not only for the above domains but also for any Riemann surface $\notin O_G$. The extensions of Fatou and Beurling's theorems express the behaviour of analytic functions on almost all boundary points but have no effect on the small set, $\{p\in\Delta_1(M): \delta(p)>\delta\}$. The purpose of this paper is to study analytic functions on the small set, to extend theorems in the previous one and to show some examples. Let G be a domain in a Riemann surface R. Through this paper we suppose ∂G consists of at most a countably infinite number of analytic curves clustering nowhere in R. The following lemma is useful.

LEMMA 5[2]. *Let R be a Riemann surface $\in O_g$ and let G be a domain and $U_i(z)$ $(i=1,2,\cdots,i_0)$ be a harmonic function in G such that $D(U_i(z))<\infty$. Then there exists a sequence of curves $\{\Gamma_n\}$ in R such that Γ_n separates a fixed point p_0 from the ideal boundary, $\Gamma_n\to$ideal boundary of R and $\int_{\Gamma_n\cap G}\left|\frac{\partial}{\partial n}U_i(z)\right|ds\to 0$ as $n\to\infty$ for any i.*

Generalized Gree's function[2] (abbreviated by G.G.). Let R be a Riemann surface with an exhaustion $\{R_n\}$ $(n=0,1,2,\cdots)$ and G be a domain in R. Let $w_{n,n+i}(z)$ be a harmonic function in $R_{n+i}-(G\cap(R_{n+i}-R_n))$ such that $w_{n,n+i}(z)=0$ on $\partial R_{n+i}-G$ and $=1$ on $G\cap(R_{n+i}-R_n)$. We call $\lim_n \lim_i w_{n,n+i}(z)$ a H.M. (harmonic measure) of the boundary determined by G