Rational approximation with $C(\partial K)=R(\partial K)$

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1. Introduction. Recently many suthors have studied rational approximations by various methods [3, 7, 8, etc.].

Let K be a compact subset of the complex plane $\boldsymbol{C}$, let U be the interior of K, $\mathring{K}=U$, and let ∂K be the topological boundary of K. Let $C(K)$ be the algebra of all complex-valued continuous functions on K, let $A(K)$ be the algebra of all continuous functions on K, analytic in U, and let $R(K)$ be the uniform closure of rational functions with poles off K. By $H^\infty(U)$ we denote the algebra of all bounded analytic functions on U, and by H a subalgebra of $H^\infty(U)$ which is pointwise boundedly closed on U. In this paper, we will consider rational approximations under the condition: $C(\partial K)=R(\partial K)$. First, we will give a sufficent condition under which $H\cap C(K)=R(K)$. This result is an extension of Theorem 4 in [9] in some sense. Next, by proving that the set of non-peak points for $R(K)$ is included in $\bar{U}$, the closure of U, we will obtain two conditions each of which is equivalent to the coincidence of $R(K)$ and $A(K)$. Our main tool is A. M. Davie's theorem [2] on pointwise bounded closure.

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2. Notations. All norms will be supremum norms. By measures we mean finite regular complex Borel measures and all measures considered will be supported on compact plane sets. By an annihilating measures for $R(K)$ we mean a measure τ on K satisfying

$$\int_K f\,d\tau=0 \quad \text{for all} \quad f\in R(K).$$

We denote the set of annihilating measures by $R(K)^\perp$. If $w\in K$ we define a positive or complex representing measure of w for $R(K)$ to be a measure ν on K satisfying

$$f(w)=\int_K f\,d\nu \quad \text{for all} \quad f\in R(K).$$

Let σ be a positive measure on K. By $H^\infty(\sigma)$ we denote the weak-star closure of $R(K)$ in $L^\infty(\sigma)$. A point w in K is called a peak point if there