Kaehler submanifolds with $RS=0$ in a complex projective space

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P. J. Ryan [3] and T. Takahashi [4] has recently studied complex hypersurfaces in a complex space form satisfying the condition

$$R(X, Y)S=0 \tag{0.1}$$

for any vectors X and Y of the hypersurface, where R denotes the curvature tensor, S is the Ricci tensor and $R(X, Y)$ operates on the tensor algebra as a derivation. Ryan proved that these hypersurfaces are Einstein if the ambient space is not complex euclidean, which was generalized by M. Kon [1] in the case of Kaehler submanifolds in a complex space form of constant negative holomorphic sectional curvature. On the other hand, Takahashi also verified that such hypersurfaces are cylindrical if the ambient space is complex euclidean.

The purpose of this note is to prove the following

THEOREM. *Let M be an n-dimensional Kaehler submanifold immersed in an $(n+q)$-dimensional complex projective space PC_{n+q}. If M satisfies the condition* (0.1) *and the codimension q is less than $n-1$, then M is Einstein.*

§1. Kaehler submanifolds in PC_{n+q}

Let M be an n-dimensional Kaehler manifold and ι an isometric and holomorphic immersion of M into an $(n+q)$-dimensional complex projective space $P_{n+q}(c)$ of constant holomorphic sectional curvature c. We call such ι simply a *Kaehler immersion*. Throughout this note, M may be identified with $\iota(M)$, because the argument is local. Let $e_1, \cdots, e_n, e_{n+1}, \cdots, e_{n+q}$ be a unitary frame field in $P_{n+q}(c)$ in such a way that, restricted to M, $e_1, \cdots, e_n$ are tangent to M. Its dual coframe field $\omega^1, \cdots, \omega^n, \omega^{n+1}, \cdots, \omega^{n+q}$ consists of complex valued linear differential forms of type $(1, 0)$ on M such that

$$\omega^\alpha=0, \tag{1.1}$$

and $\omega^1, \cdots, \omega^n, \omega^{-1}, \cdots, \omega^{-n}$ are linearly independent. Greek indices run over the range $n+1, \cdots, n+q$. The induced Kaehler metric g on M is given