On projective *H*-separable extensions

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Introduction

All notations and terminologies in this paper are same as those in the author's previous papers [7], [8], [9], [10] and [11]. All rings shall have identities, and all subrings of them shall have the same identities as them. Whenever we denote a ring and its subring by A and Γ, respectively, we shall always denote the center of A by C and the centralizers of Γ in A, i.e., $V_A(\Gamma)$, by Δ. A ring A is an *H*-separable extension of a subring Γ if $A \otimes_\Gamma A$ is A-A-isomorphic to a A-A-direct summand of a finite direct sum of copies of A. Some equivalent conditions and fundamental properties have been researched in [3], [4] and [7]. In case Γ is the center of A, this definition is same as that of Azumaya algebra, and we have found in *H*-separable extension many similar properties to Azumaya algebra. In §1 we shall study in what case an *H*-sparable extension A of Γ become Γ-projective. If B is an intermediate subring of A and Γ such that $_BB_\Gamma < \oplus\ _BA_\Gamma$ and B is left relatively separable over Γ in A, A is left B-projective. And if furthermore B is right relatively separable over Γ in A, A is a left QF-extension of B (Theorem 1.1). In §2 we shall study some relations between *H*-separable extensions of simple rings and classical fundamental theorem on simple rings. The latter states that if A is a simple ring with its center C, and if D is a simple C-algeba ($[D:C]<\infty$) contained in A, then $\Gamma = V_A(D)$ is simple, $D = V_A(\Gamma)$, and some interesting commutor theorems hold in this case (see [2]). Now we shall prove that A is an *H*-separable extension of Γ in this case (Theorem 2.1). We have already found that similar commutor theorems hold in general *H*-separable extensions (see Theorem 1 [6]). In §3 we shall study some properties of ideals in *H*-separable extensions. Especially, we will see in Theorem 3.2 that if A is an *H*-separable extension of Γ such that A is right Γ-projective and a right Γ-generator, there exists a 1–1 correspondence between the class of left ideals of Γ and the class of left ideals of A which ars also right Δ-submodules.