# Some remarks on $p$-absolutely summing operators

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## §1. Introduction

In [1], Cohen has shown that the following:

THEOREM A. *Let $E$ be a normed space. Then $E$ is an inner product space iff for all Banach spaces $F$ and for all 2-absolutely summing operators $T$ mapping $E$ into $F$, the conjugate operator $T^*$ is 2-absolutely summing and* $\Pi_2(T^*) \leqq \Pi_2(T)$.

In [2], Kwapień has given a similar characterization of spaces isomorphic to inner product spaces. That is the following:

THEOREM B. *Let $E$ be a Banach space, then the following conditions are equivalent*:

(1) *$E$ is isomorphic (=linearly homeomorphic) to an inner product space.*

(2) *If $T \in \Pi_2(E, l_2)$, then $T^* \in \Pi_2(l_2, E^*)$.*

Theorem A and Theorem B suggest the following (*):

(*) *Let $E$ be a Banach space, and $1 \leqq p < \infty$. Then the following conditions are equivalent.*

(1) *For all Banach spaces $F$,*

*if $T \in \Pi_p(E, F)$, then $T^* \in \Pi_p(F^*, E^*)$.*

(2) *If $T \in \Pi_p(E, l_p)$, then $T^* \in \Pi_p(l_{p^*}, E^*)$.*

In this paper, we shall prove this fact is true, and furthermore, using weakly $p$-summable sequences, we shall characterize Banach spaces $E$ which satisfy the condition (1) (or equivalently condition (2)).

**Notation.** Throughout the paper $E$ and $F$ will denote Banach spaces and $E^*$ and $F^*$ the continuous dual spaces. The space of continuous linear operators mapping $E$ into $F$ will be denoted by $L(E, F)$.

## §2. Basic definitions and well known results

Let $E$ and $F$ be Banach spaces, and $1 \leqq p \leqq \infty$.

A sequence $\{x_i\}$ with values in $E$ is called weakly $p$-summable ($l_p(E)$) if for all $x^* \in E^*$, the sequence $\{x^*(x_i)\} \in l_p$. The space $l_p(E)$ is a normed