Some remarks on local solvability and hypoellipticity of second-order abstract evolution equations

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0. Introduction

In this paper we shall consider local solvability and hypoellipticity of the following operator:

$$P=(\partial_t+at^kA)(\partial_t+bt^kA)-ct^{k-1}A\,. \tag{0.1}$$

Here A is a selfadjoint operator in a Hilbert space H, a, b, c are complex numbers and k a positive odd integer. Namely we shall prove the following

THEOREM. *Assume that* Re $a>0$ *and* Re $b<0$, *then the following are equivalent*:

(i) *P is locally solvable at $t=0$;*
(ii) *P is hypoelliptic at $t=0$;*
(iii) *for no integer n, $c/(a-b)=1-n(k+1)$ or $-n(k+1)$.*

The case when $A=D_x$ and $H=L^2(\boldsymbol{R}^1)$ or when A positive-definite and $k=1$ under more general set-up were considered by Gilioli and Treves [3] or Treves [6] respectively (see also [2] and [5]). For the case in which k is even, refer to Menikoff [4].

In [3] the sufficiency proof for local solvability was based on the theory of ordinary differential operators of Fuchs' type (see also [1]). However, in the present paper we shall assert that it can be proved in the framework of abstract theory in [6].

1. Preliminary

In order to describe the situation more precisely we here explain our notations and list up some results in [6] and [3] which we must use.

Let A be a densely defind linear operator on H. We shall assume that A is selfadjoint and that $(I-E(0))A$ and $E(0)A$ are unbounded. Here we denote the spectral resolution of A by $E(\lambda)$. (cf. Yosida [7]) For some $\varepsilon>0$, we define an orthogonal decomposition of H;

$$H_+=\big(I-E(\varepsilon)\big)H, \qquad H_\varepsilon=\big(E(\varepsilon)-E(-\varepsilon)\big)H, \qquad H_-=E(-\varepsilon)H\,.$$