Parametrices for pseudo-differential equations with double characteristics I.

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0. Introduction.

In this paper we shall construct parametrices and prove solvability and hypoellipticity for some classes of pseudo-differential operators whose characteristic set is a closed manifold of codimension 2 in the cotangent space.

We shall consider a pseudo-differential operator $L(x, D)$ with double characteristics ;

$$L(x, D)=(P\circ Q)(x, D)+R(x, D),$$

where P, Q and R are pseudo-differential operators whose principal part are essentially transformed into the pseudo-differential operator of the following type (M) $D_n - i x_n^k\ a(x, D')$ (with $a(x, \xi')\neq 0$) by a canonical transformation.

Theory of the local solvability of pseudo-differential operators with simple characteristics was extensively studied in [1] and [14]. The description of their condition was based on the classical Hamilton-Jacobi theory of characteristics and bicharacteristics. However, the case of multiple characteristics is much more complicated. The good example of pseudo-differential operators with double characteristics are pseudo-differential operators whose principal symbols are written by the product of those of the type (M).

Investigation of the operator whose principal part is the product of abstract first order evolutional equations was first made in Treves [18] when $k=1$. Furthermore, for general odd integer k, Gilioli-Treves [7] obtained the necessary and sufficient conditions of local solvability for the differential operator R^2 whose principal part is the product of the differential operators of the type (M) with $a(x, D')=aD'$. However, when the base space is a manifold, their conditions was not intristic for coordinate systems and in particular, it is not clear how to microlocalize the pseudo-differential operator in their argument. As a matter of fact, in the special case when $k=1$ and the base space is more general manifold, Boutet de