On some variational properties of submanifolds in Riemannian spaces

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§1. Introduction. Let V^n be a closed orientable hypersurface in an $(n+1)$-dimensional Euclidean space E^{n+1} and $\bar{V}^n(\varepsilon)$ be a family of admissible hypersurfaces parameterized by the real number ε near $\varepsilon=0$ such that $\bar{V}^n(0)=V^n$. We put

$$J[H_1^c]=\int_{V^n} H_1^c\, d\sigma\,,$$

where c is an arbitrary positive integer, H_1 is the mean curvature of V^n and $d\sigma$ means the volume element of V^n. We denote by δJ the first variation of the functional J:

$$\delta(J[H_1^c])=\left(\frac{\partial}{\partial\varepsilon}J[\bar{H}_1^c(\varepsilon)]\right)_{\varepsilon=0}\,,$$

where $\bar{H}_1(\varepsilon)$ is the mean curvature of $\bar{V}^n(\varepsilon)$.

The normal variation is defined to be the variation such that the direction of the deformation at each point of V^n is in the direction of the normal of V^n. V^n is said to be stable with respect to $J[H_1^c]$ if $\delta(J[H_1^c])=0$ for any normal variation. In particular, when V^n is stable with respect to $J[H_1^n]$, V^n is called the stable hypersurface. B. Y. Chen [1][1)] has proved that a closed orientable hypersurface V^n in E^{n+1} is stable with respect to $J[H_1^c]$ if and only if H_1 and R' satisfy

$$c\Delta H_1^{c-1}+n^2(c-1)H_1^{c+1}+cH_1^{c-1}R'=0\,, \tag{1.1}$$

where Δ denotes the Laplacian with respect to the induced metric on V^n and R' is the scalar curvature of V^n. When $c=1$, we obtain from (1.1), $R'=0$ and this result was given by M. Pinl and H. W. Trapp [2]. If we denote by H_2 the second mean curvature of V^n, from the Gauss equation we get

$$R'=-n(n-1)H_2\,. \tag{1.2}$$

Therofore we can see that if a closed orientable hypersurface V^n in E^{n+1} is stable with respect to $J[H_1]$, then $H_2=0$.

1) Numbers in brackets refer to the references at the end of the paper.