# $U$-rational extension of a ring

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## Introduction.

Let $R$ be a ring with identity and $U$ be a right $R$-module such that $R\subset\Pi E(U)=C$ where $E(U)$ is the injective hull of $U$. Then the double centralizer of $C$ is a ring $S$ and is a $U$-rational extension of $R$ as a right $R$-module. A ring $S$ is regarded as a subring of a maximal right quotient ring of $R$.

In [5], K. Masaike states a characterization of a ring of which a canonical inclusion of $R$ into a maximal quotient ring is a right flat epimorphism. We will generalize this result for a canonical inclusion of $R$ into $S$.

Throughout this paper, a ring $R$ has always an identity element and an $R$-module is unital. An injective hull of an $R$-module $M$ is written by $E(M)$. Let $X$ and $Y$ be the right $R$-modules. We say $X$ is $Y$-torsionless if $X$ is embeddable into some product of $Y$, i.e., $X\subset\Pi Y$. This is equivalent that for any nonzero $x\in X$ there exists an $R$-homomorphism $f$ of $X$ into $Y$ such that $f(x)\neq 0$.

## 1. $U$-rational extension of a ring

Let $U$ be a right $R$-module such that $E(U)$ is faithful. Then we have $R\subset\Pi E(U)$. We put $C=\Pi E(U)$, $H=\mathrm{Hom}_R(C, C)$. Then $C$ becomes a bimodule ${}_HC_R$, thus we get $S=\mathrm{Hom}_H(C, C)$ the double centralizer of $C_R$.

PROPOSITION 1. *$C$ is injective as a right $S$-module,* $\mathrm{Hom}_R(C, C)=\mathrm{Hom}_S(C, C)$, *and if $B_R$ is a direct summand of $C_R$, then $B$ is a right $S$-module and also a direct summand of $C$ as a right $S$-module.*

PROOF. This is well-known (see [3], [4] for example), but for the completeness, we state the proof.

Let $0\rightarrow X\rightarrow Y$ be an exact sequence of right $S$-modules, and $f$ be an $S$-homomorphism of $X$ into $C$. Since $C_R$ is injective, $f$ can be extended to $g: Y_R\rightarrow C_R$. We will show that $g$ is an $S$-homomorphism.

For any $y\in Y$, define the mapping $k_y: S\rightarrow C$ by $k_y(s)=g(ys)-g(y)s$ for $s\in S$. This is clearly an $R$-homomorphism and can be extended to $k'_y\in H$ by injectivity of $C_R$. Then $k'_y(R)=k_y(R)=0$, therefore, $k_y(s)=k'_y(s)=k'_y((1)s)=(k'_y(1))s=0$ (here we use the canonical embedding of $S_R$ into $C_R$; $s\mapsto(1)s$).