Invariant hypersurfaces of $S^n \times S^n$ with constant mean curvature

Yoshio MATSUYAMA

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0. Introduction

Ludden and Okumura [1] studied minimal hypersurfaces of the product $S^n \times S^n$ of two n-spheres. Some of their results are as follows:

a. If a compact orientable minimal hypersurface M of $S^n \times S^n$ $(n>1)$ satisfies

$$\int_M (S^2-(n-1)S)\,dM \geqq \int_M \|\nabla H\|^2 dM$$

(in particular, $\nabla H=0$ and $S\geqq n-1$), then the tangent space of M is invariant under an almost product structure on $S^n \times S^n$ (for simplicity, we say that M is an invariant hypersurface), where $S=\text{trace } H^2$.

b. Let M be a compact orientable invariant minimal hypersurface of $S^n \times S^n$. Then either M is the totally geodesic hypersurface or $S\equiv n-1$, or $S(x)>n-1$ at some $x\in M$.

c. $S^{n-1}(1)\times S^n(1)$ and

$$S^m(\sqrt{m/(n-1)})\times S^{n-m-1}(\sqrt{(n-m-1)/(n-1)})\times S^n(1)$$

are the only compact orientable invariant minimal hypersurfaces of $S^n \times S^n$ satisfying $S\leqq n-1$.

In the present paper, we further investigate hypersurfaces of $S^n \times S^n$ under the assumption of non-negative sectional curvature.

That is, we obtain the following results:

A. A compact orientable minimal hypersurface with non-negative sectional curvature of $S^n \times S^n$ $(n>1)$ which satisfies

$$\int_M (S^2-(n-1)S)\,dM \geqq 0$$

(in particular, $S\geqq n-1$) is an invariant hypersurface (Theorem 1.2 and Corollary 1.3).

B. Let M be a compact orientable invariant minimal hypersurface with non-negative sectional curvature of $S^n \times S^n$. Then either M is the totally geodesic hypersurface or $S\equiv n-1$ (Theorem 2.1).