# Characterizations of the topology of uniform convergence on order-intervals

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## 1. Introduction

Nakano's theorem [6], which states that a boundedly and locally order complete topological vector lattice $(E, C, \mathscr{T})$ is complete, is one of the deepest results in the theory of topological vector lattices. It is known from [16, (13.9)] that the converse of Nakano's theorem holds for the topology $o(E, E')$ of uniform convergence on all order-intervals in $E'$. Therefore it is interesting to find some characterizations for the topology $\mathscr{T}$ to be $o(E, E')$ where $(E, C, \mathscr{T})$ is only assumed to be a locally solid space. The purpose of this paper is to give such characterizations in terms of some special continuous linear mappings.

Definitions and some remarkable properties of ordered sequence vector spaces, which we shall need in what follows, are explained in section 2.

Cone-absolutely summing mappings were first considered by Schaefer [12] and Schlotterbeck in the Banach lattice case, and were extended by Walsh [13] to the case of locally solid spaces. Using a characterization of cone-absolutely summing mappings defined on a locally solid space $(E, C, \mathscr{T})$, we obtain a necessary and sufficient condition for $\mathscr{T}$ to be $o(E, E')$. A connection between the nuclearity and the topology $\sigma(E, E')$ is given by Theorem 3.7.

In section 4 we define $L$-prenuclear seminorms in terms of the notion of cone-absolutely summing mappings, and then it is shown that $\mathscr{T}=o(E, E')$ if and only if each continuous seminorm is $L$-prenuclear. On the other hand, it is known from Schaefer [12, p. 178] that the notion of prenuclear seminorm is useful for the investigation of nuclearity. A connection between prenuclear seminorms and absolutely summing mappings is given in this section.

$L$-nuclear mappings are defined by means of $L$-prenuclear seminorms. Another characterization of $\mathscr{T}$ to be $o(E, E')$ is given in terms of $L$-nuclear mappings. In particular, the identity map is $L$-nuclear if and only if $\mathscr{T}=o(E, E')$ and $E'$ has an order unit. It is amusement to compare this result with the Dvoretzky and Rogers theorem. Similarly prenuclear linear