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A certain congruence theorem for closed submanifolds of codimension 2 in a space of constant curvature

(The continuation of the previous paper [1][1])

Dedicated to Professor Tominosuke Ōtuki on his 60th birthday

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Introduction.

We consider an $(m+2)$-dimensional orientable Riemannian space S^{m+2} with constant curvature of class $C^{\nu}(\nu\geqq 3)$ which admits a continuous differentiable one-parameter group G of $1-1$ mappings T_{τ} of S^{m+2} onto itself (the group parameter τ, $-\infty<\tau<+\infty$, is assumed to be always canonic i.e., $T_{\tau_1}\cdot T_{\tau_2}=T_{\tau_1+\tau_2}$). We assume that the orbits (or streamlines) of the points of S^{m+2} produced by T_{τ} are regular curves covering S^{m+2} simply.

The purpose of the present paper is to generalize the following theorem given by H. Hopf and the present author [2] for two orientable closed submanifolds of codimension 2.

THEOREM. *Let W^{m+1} and $\overline{W}^{m+1}$ be two orientable closed hypersurfaces in S^{m+2} and p and $\bar{p}$ be the corresponding points of these hypersurfaces along an orbit, and $H_r(p)$ and $\overline{H}_r(p)$ be the r-th mean curvature at these points respectively. Assume that the set of points in which the orbit is tangent to W^{m+1} or $\overline{W}^{m+1}$ has no inner point and that the second fundamental form of $W^{m+1}(t)\overset{\mathrm{def}}{=}(1-t)W^{m+1}+t\overline{W}^{m+1}$, $0\leqq t\leqq 1$ is positive definite. If G is a group of isometries of S^{m+2} and if the relation $H_r(p)=\overline{H}_r(p)$ holds for each point $p\in W^{m+1}$, then W^{m+1} and $\overline{W}^{m+1}$ are congruent mod G.*

Especially, in case of $r=m$, that is, the generalized theorem relating to the Gauss curvature was already proved in the previous paper [1].

§1. Generalized theorem.

We suppose an $(m+2)$-dimensional orientable Riemannian space S^{m+2} with constant curvature of class $C^{\nu}(\nu\geqq 3)$ which admits an infinitesimal isometric transformation

$$\hat{x}^i = x^i + \xi^i(x)\,\delta\tau \tag{1.1}$$

1) Numbers in brackets refer to the references at the end of the paper.